Fourier Series & Frequency Domain

https://en.wikipedia.org/wiki/Fourier_transform
Input Spectrum
Application: Digital Communication
Applying Superposition

![Graph showing a periodic voltage waveform over time in nanoseconds. The voltage is represented on the y-axis (V) and time on the x-axis (ns). The waveform consists of a series of square pulses.](image-url)
Calculated Output Voltage

Graph showing the calculated output voltage over time.
Simulation Verification

\begin{align*}
V_{in} \text{ [V]} & \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \quad 1.2 \quad 1.4 \\
V_{out} \text{ [V]} & \quad 0.57 \quad 0.58 \quad 0.59 \quad 0.6 \quad 0.61 \quad 0.62 \quad 0.63 \\
\end{align*}

V(in) \quad V(out)

\begin{align*}
\text{time [ns]} & \quad 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \\
\text{PULSE(0 1.2 0 1p 1p 125p 250p)} \\
\text{.tran 0 1u \{1u-1.25n\}}
\end{align*}
Frequency Domain Interpretation

| | \( |H(jw)| \) | \( F[V_{in}] \) | \( F[V_{in}]^*|H(j\omega)| \) |
|---|---|---|---|
| frequency \([\text{Hz}]\) (Log Scale) | \( 10^0 \) | \( 10^{-5} \) | \( 10^0 \) |
| \( |H(jw)| \) (Log Scale) | \( 10^{-5} \) | \( 10^0 \) | \( 10^{-5} \) |
| \( F[V_{in}] \) | \( 10^0 \) | \( 10^0 \) | \( 10^0 \) |
| \( F[V_{in}]^*|H(j\omega)| \) | \( 10^0 \) | \( 10^0 \) | \( 10^0 \) |

The graphs show the magnitude and phase response of a system in the frequency domain, with frequency on a log scale.
Complex Form of Fourier Series
Fourier Series Representation

Assume we have some function $f(t)$ which is periodic with period $T_0 = \frac{2\pi}{\omega_0}$

$$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

$$a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \cos(k\omega_0 t) dt$$

$$b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} f(t) \sin(k\omega_0 t) dt$$

$f(t)$ can be expressed this way if

1. $f(t)$ is single-valued
2. $\int_{t_0}^{t_0+T_0} |f(t)| dt$ exists
3. $f(t)$ had finite discontinuities and max/min per period

Alternate forms

$$f(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k)$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

$$\varphi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{2}(a_k - jb_k)$$

$$c_{-k} = \frac{1}{2}(a_k + jb_k)$$

$$c_0 = a_0$$

$$c_k = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} f(t)e^{-jk\omega_0 t} dt$$
Non-periodic Waveforms: Fourier Transform