Fourier Series of a Pulse Train

\[ a_0 = A \frac{\tau}{T} \]

\[ b_k = 0 \]

\[ a_k = \frac{2A}{k\pi} \sin\left(k\pi \frac{\tau}{T}\right) \]

\[ c_k = \frac{A}{k\pi} \sin\left(k\pi \frac{\tau}{T}\right) = \frac{a_k}{2} \]

\[ \frac{1}{x} \sin(x) = \text{sinc}(x) \]

Example 17.4 in back
Example Matlab Calculation

\[ f = 200 \text{ Hz} \]
\[ T = 5 \text{ ms} \]
\[ \tau = 2 \text{ ms} \]

\[ D = \frac{\tau}{T} \]

---

**Fourier Series Approx**

\[
f = 200; \\
A = 1; \\
\tau = 2e-3; \\

t = linspace(-1/f*5,1/f*5,100000); \\
a0 = A*\tau*f; \\
sum = a0*(t./t); \\
kmax = 200; \\
for \ k=1:kmax \\
    ak(k) = 2*A/k/pi*sin(k*pi*D); \\
    sum = sum + ak(k)*cos(k*\omega0*t); \\
end
\]
Example Matlab Calculation

\( f = 100 \text{ Hz} \)

\( T = 10 \text{ ms} \)

\( \tau = 2 \text{ ms} \)
Example Matlab Calculation

\[ f = 50 \text{ Hz} \]
\[ T = 20 \text{ ms} \]
\[ \tau = 2 \text{ ms} \]
Example Matlab Calculation

\[ f = 10 \text{ Hz} \]
\[ T = 100 \text{ ms} \]
\[ \tau = 2 \text{ ms} \]
Alternate View

\( f = 100 \text{ Hz} \)
\( T = 10 \text{ ms} \)
\( \tau = 2 \text{ ms} \)

\( f = 10 \text{ Hz} \)
\( T = 100 \text{ ms} \)
\( \tau = 2 \text{ ms} \)
Non-periodic Waveforms: Fourier Transform

Fourier Series → only periodic waveforms
Fourier Transform → non-periodic signals
→ treat any non-periodic signal as a periodic signal with $T \to \infty$

Fourier Series:
\[
C_k = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j2\pi k t/T} dt
\]

Fourier Transform:
\[
F(k) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi k t} dt = F(k)
\]

Fourier Series:
\[
f(t) = C_0 + \sum_{k=\infty}^{\infty} C_k e^{j2\pi k t/T}
\]

Fourier Inverse Transform:
\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k) e^{j2\pi k t/T} dt
\]
Fourier Series of Impulse Train

\[ f = 10 \text{ Hz} \]
\[ T = 100 \text{ ms} \]
\[ \tau = 2 \text{ ms} \]

\[ f = 1000 \text{ Hz} \]
\[ T = 1 \text{ ms} \]
\[ \tau = .02 \text{ ms} \]
Applications of Fourier Transform

• Imaging
  – Spectroscopy, x-ray crystallography
  – MRI, CT Scan
• Image analysis
  – Compression
  – Feature extraction
• Signal processing
  – Audio filtering
  – Spike detection
  – Modeling sampled systems (A/D & D/A)
  – Understanding aliasing
  – Speech recognition
• RF Communications
  – AM & FM Encoding

https://youtu.be/r6sGWTCMz2k?t=1310
Chapter 14

S-DOMAIN CIRCUIT ANALYSIS
Transform Domains

**Phasor Transform**

\[ u(t) = A\cos(\omega_0 t + \varphi) \]

- Circuit with inductor(s) and capacitor(s)
  - Phasor Transform
  - Phasor-domain circuit
  - Algebraic Eqs
  - Phasor-domain solution
  - Inverse phasor Transform
  - Sinusoidal steady-state solution

**Fourier Transform**

\[ u(t) = \sum C_n \cos(n\omega_0 t + \varphi_n) \]

- Circuit with inductor(s) and capacitor(s)
  - Fourier Transform
  - Frequency-domain circuit
  - Algebraic Eqs
  - Frequency-domain solution
  - Inverse Fourier Transform
  - steady-state solution

**Laplace Transform**

\[ u(t) = \sum K_n e^{s_n t} \]

- Circuit with inductor(s) and capacitor(s)
  - Laplace Transform
  - s-domain circuit
  - Algebraic Eqs
  - s-domain solution
  - Inverse Laplace Transform
  - solution
The Laplace Transform

Take Fourier Transform & replace \((\nu)\) w/ \(s = \sigma + j\omega\)

\[
F(s) = \int_0^\infty e^{-st} f(t) \, dt
\]

\[
f(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} \, ds
\]

Usually (always in ECE 202) we'll use the unilateral Laplace Transform

\[
F(s) = \int_0^\infty e^{-st} f(t) \, dt
\]

\[
f(t) = \frac{1}{2\pi j} \int_{j\infty}^{-j\infty} F(s) e^{st} \, ds
\]

Short-hand:

\[
F(s) = L\{f(t)\} = L\{f(t)\}
\]

\[
f(t) = L^{-1}\{F(s)\} = L^{-1}\{F(s)\}
\]

Time-domain

- \(f(t)\) signals
- ODEs systems

Frequency/Fourier Domain

- \(F(j\omega)\) signals
- \(H(j\omega)\) systems

Laplace/s/complex freq Domain

- \(F(s)\) signals
- \(H(s)\) systems