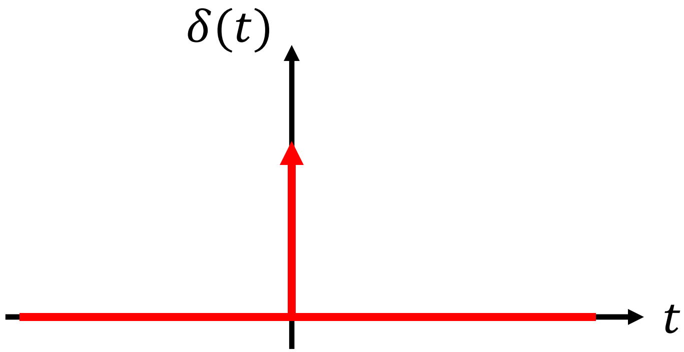
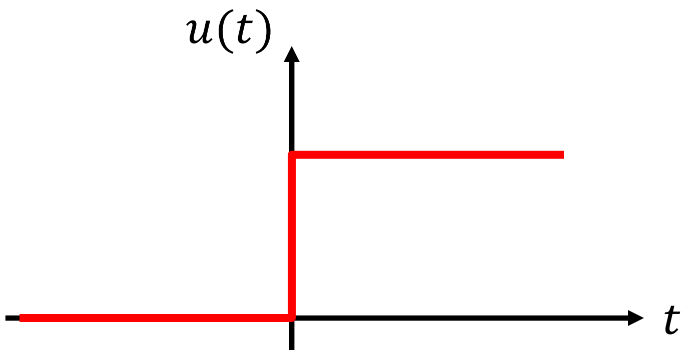
Complex Frequency

Impulse, Step, and Ramp Functions

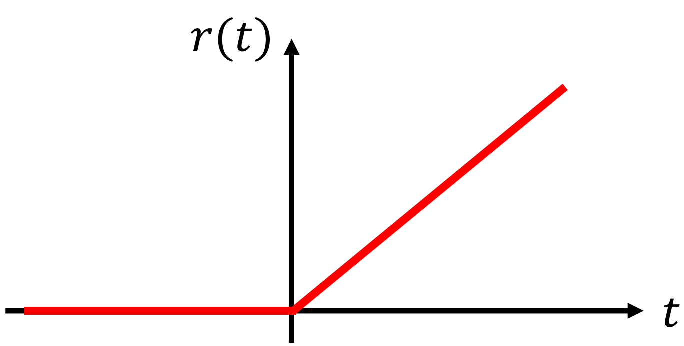


$$\delta(t)\begin{cases}0 & t \neq 0\\ \infty & t = 0\end{cases}$$

$$\int_{-\infty}^{+\infty}\delta(t)dt = \int_{0^-}^{0^+}\delta(t)dt = 1$$



$$u(t)\begin{cases}0 & t < 0\\ 1 & t > 0\end{cases}$$



$$r(t) = tu(t) = \begin{cases}0 & t \leq 0\\ t & t \geq 0\end{cases}$$

Sifting Property of Impulse Function $\delta(t)$

Example Signal Laplace Transforms

TABLE 14.1 Laplace Transform Pairs

$f(t) = \mathscr{L}^{-1}\{\mathbf{F}(\mathbf{s})\}$	$\mathbf{F}(\mathbf{s}) = \mathscr{L}\{f(t)\}$	$f(t) = \mathscr{L}^{-1}\{\mathbf{F}(\mathbf{s})\}$	$\mathbf{F}(\mathbf{s}) = \mathscr{L}\{f(t)\}$
$\delta(t)$	1	$\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})\,u(t)$	$\frac{1}{(\mathbf{s} + \alpha)(\mathbf{s} + \beta)}$
$u(t)$	$\frac{1}{\mathbf{s}}$	$\sin \omega t\, u(t)$	$\frac{\omega}{\mathbf{s}^2 + \omega^2}$
$tu(t)$	$\frac{1}{\mathbf{s}^2}$	$\cos \omega t\, u(t)$	$\frac{\mathbf{s}}{\mathbf{s}^2 + \omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t), n = 1, 2, \ldots$	$\frac{1}{\mathbf{s}^n}$	$\sin(\omega t + \theta)\, u(t)$	$\frac{\mathbf{s}\sin\theta + \omega\cos\theta}{\mathbf{s}^2 + \omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{\mathbf{s} + \alpha}$	$\cos(\omega t + \theta)\, u(t)$	$\frac{\mathbf{s}\cos\theta - \omega\sin\theta}{\mathbf{s}^2 + \omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(\mathbf{s} + \alpha)^2}$	$e^{-\alpha t}\sin \omega t\, u(t)$	$\frac{\omega}{(\mathbf{s} + \alpha)^2 + \omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t), n = 1,2,\ldots$	$\frac{1}{(\mathbf{s} + \alpha)^n}$	$e^{-\alpha t}\cos \omega t\, u(t)$	$\frac{\mathbf{s} + \alpha}{(\mathbf{s} + \alpha)^2 + \omega^2}$

Properties of the Laplace Transform