

TABLE 14.1 Laplace Transform Pairs

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
$\delta(t)$	1	$\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})u(t)$	$\frac{1}{(s + \alpha)(s + \beta)}$
$u(t)$	$\frac{1}{s}$	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
$tu(t)$	$\frac{1}{s^2}$	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t), n = 1, 2, \dots$	$\frac{1}{s^n}$	$\sin(\omega t + \theta)u(t)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\cos(\omega t + \theta)u(t)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^2}$	$e^{-\alpha t} \sin \omega t u(t)$	$\frac{\omega}{(s + \alpha)^2 + \omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t), n = 1, 2, \dots$	$\frac{1}{(s + \alpha)^n}$	$e^{-\alpha t} \cos \omega t u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega^2}$

Properties of the Laplace Transform



Initial and Final Value Theorems

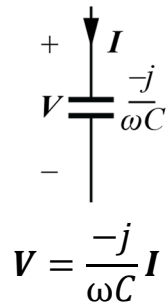
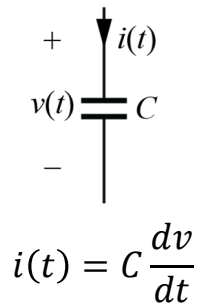
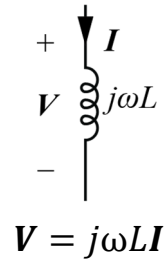
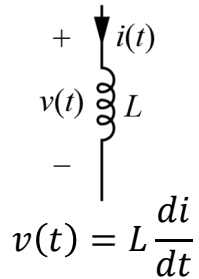
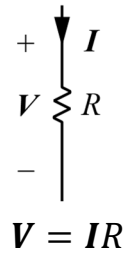
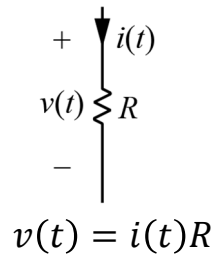
Operation	$f(t)$	$\mathbf{F}(s)$
Addition	$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(s) \pm \mathbf{F}_2(s)$
Scalar multiplication	$kf(t)$	$k\mathbf{F}(s)$
Time differentiation	$\frac{df}{dt}$	$s\mathbf{F}(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2\mathbf{F}(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3\mathbf{F}(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time integration	$\int_{0^-}^t f(t) dt$	$\frac{1}{s}\mathbf{F}(s)$
	$\int_{-\infty}^t f(t) dt$	$\frac{1}{s}\mathbf{F}(s) + \frac{1}{s} \int_{-\infty}^{0^-} f(t) dt$
Convolution	$f_1(t) * f_2(t)$	$\mathbf{F}_1(s)\mathbf{F}_2(s)$
Time shift	$f(t-a)u(t-a), a \geq 0$	$e^{-as}\mathbf{F}(s)$
Frequency shift	$f(t)e^{-at}$	$\mathbf{F}(s+a)$
Frequency differentiation	$tf(t)$	$-\frac{d\mathbf{F}(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^{\infty} \mathbf{F}(s) ds$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}\mathbf{F}\left(\frac{s}{a}\right)$
Initial value	$f(0^+)$	$\lim_{s \rightarrow \infty} s\mathbf{F}(s)$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} s\mathbf{F}(s)$, all poles of $s\mathbf{F}(s)$ in LHP
Time periodicity	$f(t) = f(t+nT),$ $n = 1, 2, \dots$	$\frac{1}{1 - e^{-Ts}}\mathbf{F}_1(s),$ where $\mathbf{F}_1(s) = \int_{0^-}^T f(t)e^{-st} dt$

Circuit Laplace Transform

Time Domain

Phasor Domain

s-Domain



Laplace Transform of Diff EQs

N^{th} order circuit with sinusoidal input described by ($M \leq N$ for causality)

$$b_N \frac{d^N}{dt^N} v_o(t) + \dots + b_1 \frac{d}{dt} v_o(t) + b_0 v_o(t) = a_M \frac{d^M}{dt^M} v_i(t) + \dots + a_1 \frac{d}{dt} v_i(t) + a_0 v_i(t)$$

$$\sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t)$$

Then the Laplace transform of the circuit, neglecting initial conditions, is

$$\mathcal{L} \left\{ \sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) \right\} = \mathcal{L} \left\{ \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t) \right\}$$

$$\sum_{i=0}^N b_i s^i V_o(s) = \sum_{i=0}^M a_i s^i V_i(s)$$

Rearranging:

$$\boxed{\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}}$$

Inverse Transforms