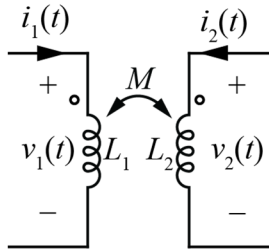


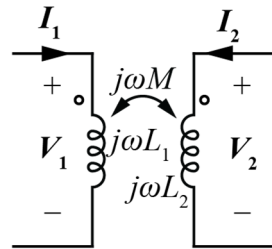
Time Domain



$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

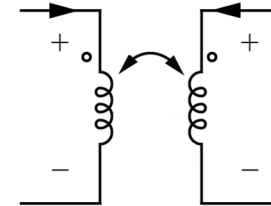
Phasor Domain



$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

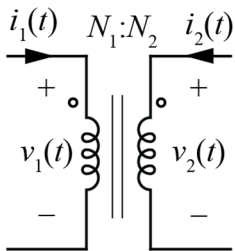
$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

s-Domain



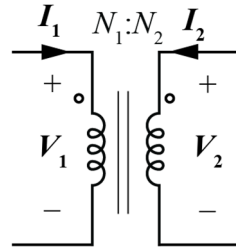
$$V_1(s) = sL_1 I_1(s) - L_1 i_1(0^-) + sM I_2(s) - M i_2(0^-)$$

$$V_2(s) = sM I_1(s) - M i_1(0^-) + sL_2 I_2(s) - L_2 i_2(0^-)$$



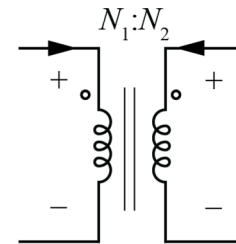
$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2}$$

$$N_1 i_1(t) + N_2 i_2(t) = 0$$



$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$$N_1 I_1 + N_2 I_2 = 0$$



$$\frac{V_1(s)}{N_1} = \frac{V_2(s)}{N_2}$$

$$N_1 I_1(s) + N_2 I_2(s) = 0$$

Laplace Transform of Diff EQs

N^{th} order circuit with sinusoidal input described by ($M \leq N$ for causality)

$$b_N \frac{d^N}{dt^N} v_o(t) + \dots + b_1 \frac{d}{dt} v_o(t) + b_0 v_o(t) = a_M \frac{d^M}{dt^M} v_i(t) + \dots + a_1 \frac{d}{dt} v_i(t) + a_0 v_i(t)$$

$$\sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) = \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t)$$

Then the Laplace transform of the circuit, neglecting initial conditions, is

$$\mathcal{L} \left\{ \sum_{i=0}^N b_i \frac{d^i}{dt^i} v_o(t) \right\} = \mathcal{L} \left\{ \sum_{i=0}^M a_i \frac{d^i}{dt^i} v_i(t) \right\}$$

$$\sum_{i=0}^N b_i s^i V_o(s) = \sum_{i=0}^M a_i s^i V_i(s)$$

Rearranging:

$$\boxed{\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}}$$

Laplace Circuit Solution Algorithm

1. Transform all sources, signals into Laplace Domain
2. Transform circuit components (including initial conditions) into Laplace Domain
3. Solve the circuit using 201 techniques

$$V_o(s) = H(s)V_i(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i}$$

4. Inverse Laplace Transform to get back to time domain

Inverse Transforms

Transfer Functions

$$H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_M)}$$

Partial Fraction Expansion / Decomposition

$$\frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_M)} = \frac{k_1}{(s - p_1)} + \frac{k_2}{(s - p_2)} + \cdots + \frac{k_N}{(s - p_N)}$$

$$\frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_M)} = \frac{k_1}{(s - p_1)} + \frac{k_2}{(s - p_2)} + \cdots + \frac{k_N}{(s - p_N)}$$

PFE: Repeated Roots

Repeated Roots: Differentiation

