MATLAB

See Example in Section 14.8 on Pg 577

- `roots`
- `residue` full PFE
- `heaviside` & `dirac` → $u(t)$ & $\delta(t)$

Control Systems Toolbox

- `tf`

Symbolic Toolbox

- `syms` t s
- `laplace` & `ilaplace`
- `pretty`
- `simplify`

Example MATLAB Script

Code

```
syms s t

vi_t = heaviside(t);
VI_s = laplace(vi_t);

H_s = 1/(s + 1);

VO_s = H_s*VI_s;

vo_t = ilaplace(VO_s)
pretty(vo_t)
```

Results

```
VI_s = 1/s

VO_s =  1/(s*(s + 1))

vo_t = 1 - exp(-t)
```

Partial Fraction Expansion / Decomposition

k_i - are "residues"

factor

PFE

$$\frac{\sum_{i=0}^{M} a_i s^i}{\sum_{i=0}^{N} b_i s^i} = \frac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)} \xrightarrow{\text{PFE}} = \frac{k_1}{(s-p_1)} + \frac{k_2}{(s-p_2)} + \cdots \frac{k_N}{(s-p_N)}$$

polynomial form — factored pole/zero form — Partial Fraction Expansion

if all poles p_i are <u>real & unique</u> & $n \geq m$

use "cover-up" method: find k_i by multiplying both sides by $\underline{(s-p_i)}$ & evaluating at $\underline{s=p_i}$

ex: find k_2

$$\left.\frac{(s-z_1)(s-z_2)\cdots(s-z_M)\cancel{(s-p_2)}}{(s-p_1)\cancel{(s-p_2)}\cdots(s-p_N)}\right|_{s=p_2} = \left.\cancel{\frac{k_1(s-p_2)}{(s-p_1)}}\right|_{s=p_2} + \frac{k_2\cancel{(s-p_2)}}{\cancel{(s-p_2)}} + \cdots \left.\cancel{\frac{k_N(s-p_2)}{(s-p_N)}}\right|_{s=p_2}$$

ex

$$F(s) = \frac{4(s+2)}{s^2+4s+3} = \frac{4(s+2)}{(s+1)(s+3)} = \frac{k_1}{s+1} + \frac{k_2}{s+3} = \frac{2}{s+1} + \frac{2}{s+3}$$

$$k_1 = \left.\frac{4(s+2)}{s+3}\right|_{s=-1} = 2$$

$$k_2 = \left.\frac{4(s+2)}{s+1}\right|_{s=-3} = 2$$

$$f(t) = \mathcal{L}^{-1}\left\{F(s)\right\}$$

$$f(t) = \left[2e^{-t} + 2e^{-3t}\right]u(t)$$

PFE: Repeated Roots

$$F(s) = \frac{N(s)}{(s-p_1)(s-p_2)^2} = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \frac{k_3}{(s-p_2)^2}$$

$$= \frac{k_1}{s-p_1} + \frac{k_2 s + (k_3 - p_2 k_2)}{(s-p_2)^2} \rightarrow \frac{As+B}{(s-p_2)^2}$$

for $k_1 \rightarrow$ no change (find with coverup method)

k_2 & $k_3 \rightarrow$ two options $\begin{cases} \text{equating coefficients method} \\ \text{differentiating method} \end{cases}$

$$F(s) = \frac{N_2(s)}{(s-p_1)(s-p_2)^3} = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \frac{k_3}{(s-p_2)^2} + \frac{k_4}{(s-p_2)^3}$$

and so on for higher-order repeated roots

Equating coefficients

ex $F(s) = \dfrac{32s(s+1)}{(s+2)(s+10)^2} = \dfrac{k_1}{s+2} + \dfrac{As+B}{(s+10)^2} = \dfrac{1}{s+2} + \dfrac{31s-50}{(s+10)^2}$

$$= \frac{1}{s+2} + \frac{31}{s+10} + \frac{-360}{(s+10)^2}$$

$$f(t) = \left[e^{-2t} + 31e^{-10t} - 360te^{-10t}\right]u(t)$$

find k_1 by normal coverup method

$$k_1 = \left.\frac{32s(s+1)}{(s+10)^2}\right|_{s=-2} \rightarrow 1$$

to find k_2: Multiply both sides by full original denominator then solve to match coefficients on powers of s

$$32s^2 + 32s = 1(s+10)^2 + (As+B)(s+2)$$
$$= s^2 + 20s + 100 + As^2 + 2As + Bs + 2B$$

$$\begin{cases} s^2: & 32 = 1 + A \longrightarrow A = 31 \\ s^1: & 32 = 20 + 2A + B \checkmark \\ s^0: & 0 = 100 + 2B \longrightarrow B = -50 \end{cases}$$

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Repeated Roots: Differentiation

ex/

$$F(s) = \frac{32s(s+1)}{(s+2)(s+10)^2} = \frac{k_1}{s+2} + \frac{k_2}{s+10} + \frac{k_3}{(s+10)^2}$$

find k_1 \$ k_3 by coverup method

$$k_1 = \left.\frac{32s(s+1)}{(s+10)^2}\right|_{s=-2} = \underline{1}$$

$$k_3 = \left.\frac{32s(s+1)}{s+2}\right|_{s=-10} = \underline{-360}$$

Differentiating method: Multiply both sides by $(s+10)^2$, then differentiate w.r.t. s before evaluating at $s=-10$

$$\frac{d}{ds}\left[\frac{k_1 \cancel{(s+10)^2}}{s+2} + k_2(s+10) + \cancel{k_3}\right]\Bigg|_{s=-10} = \frac{d}{ds}\left[\frac{32s(s+1)}{s+2}\right]\Bigg|_{s=-10}$$

$$k_2 = \left[\frac{(64s+32)(s+2) - (32s^2+32)}{(s+2)^2}\right]\Bigg|_{s=-10} = \underline{31}$$

$$\boxed{F(s) = \frac{1}{s+2} + \frac{31}{s+10} + \frac{-360}{(s+10)^2}}$$