$$\frac{1}{s^2LC + sCR + 1} = \frac{1}{\frac{s^2}{4} + s + 1} = \frac{4}{s^2+4s+4} = \frac{4}{(s+2)^2} \quad \Big| \quad \frac{1}{s^2+4} = \frac{1}{(s-2j)(s+2j)}$$

$$V_o(s) = \left(\frac{2}{s^2+4}\right)\frac{1}{s^2/4+s+1} + (-1)\frac{1}{s^2/4+s+1} + \frac{5}{s} - \frac{5}{s}\,\frac{1}{s^2/4+s+1}$$

1 PFE

PFE or manipulate

tables

2 PFE

1 $$\frac{2}{s^2+4}\,\frac{4}{(s+2)^2} = \frac{k_1}{s+2} + \frac{k_2}{(s+2)^2} + \frac{k_3}{s-2j} + \frac{k_3^*}{s+2j}$$

$$k_2 = \left.\frac{2\cdot 4}{s^2+4}\right|_{s=-2} = 1$$

$$k_1 = \left[\frac{d}{ds}\,\frac{8}{s^2+4}\right]\Bigg|_{s=-2} = \left[8\,\frac{-1}{(s^2+4)^2}\,2s\right]\Bigg|_{s=-2} = \frac{1}{2}$$

$$k_3 = \left.\frac{2}{s+2j}\,\frac{4}{(s+2)^2}\right|_{s=2j} = -\frac{1}{4} \qquad k_3^* = -\frac{1}{4}$$

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2 $$\frac{5}{s}\,\frac{4}{(s+2)^2} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{(s+2)^2}$$

$$k_1 = \left.\frac{5\cdot 4}{(s+2)^2}\right|_{s=0} = 5$$

$$k_3 = \left.\frac{20}{s}\right|_{s=-2} = -10$$

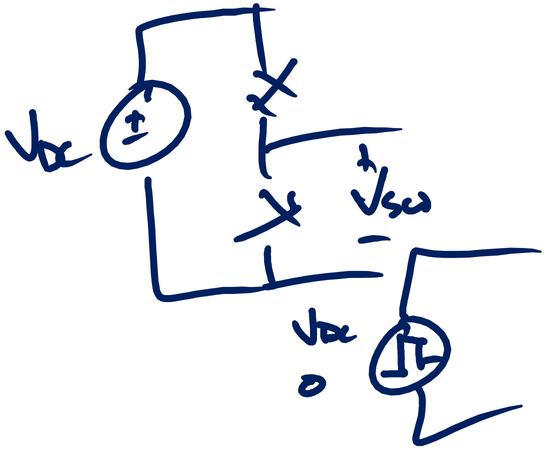
$$k_2 = \left[\frac{d}{ds}\,\frac{20}{s}\right]\Bigg|_{s=-2} = \frac{-20}{4} = -5$$

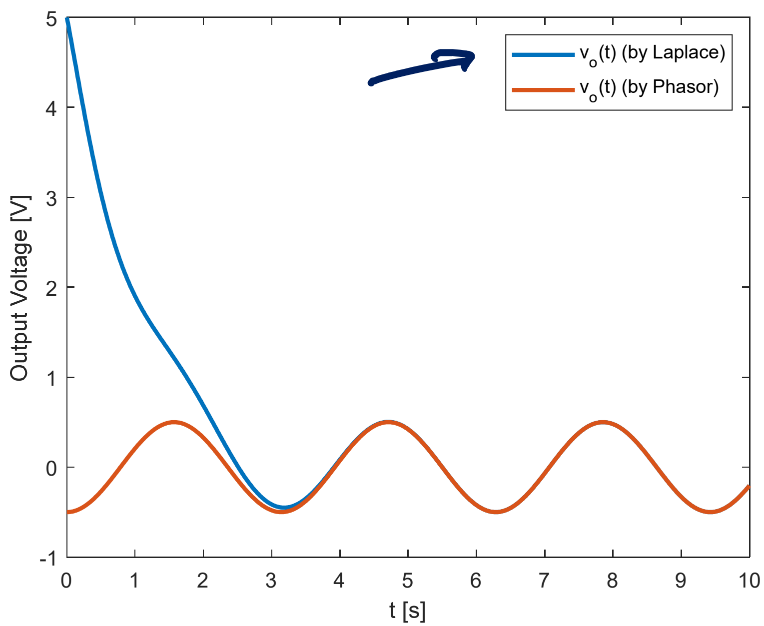
$$V_o(s) = \frac{1/2}{s+2} + \frac{1}{(s+2)^2} + \frac{-1/4}{s-2j} + \frac{-1/4}{s+2j} - \frac{4}{(s+2)^2} + \cancel{\frac{5}{s}} - \cancel{\frac{5}{s}} - \frac{-5}{s+2} - \frac{-10}{(s+2)^2}$$

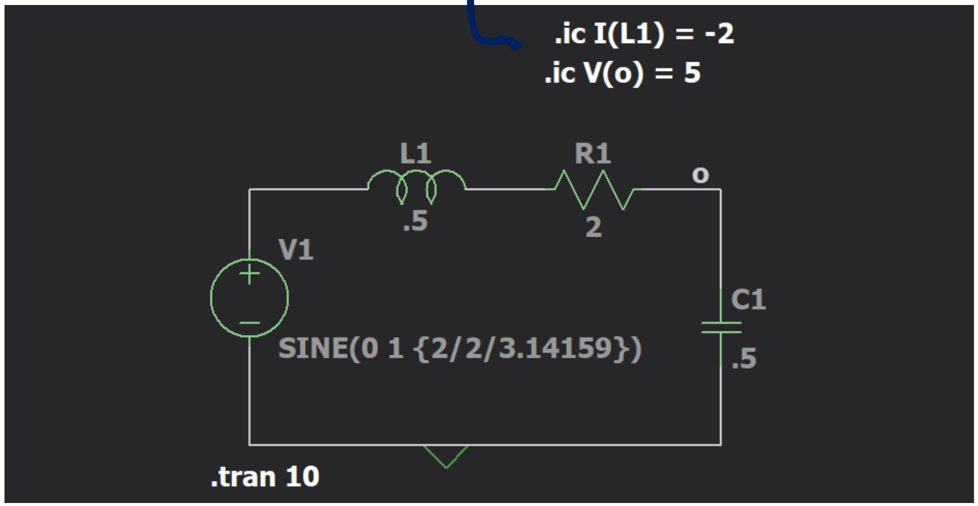
$$v_o(t) = \left[\frac{1}{2}e^{-2t} + te^{-2t} - \frac{1}{2}\cos(2t) - 4te^{-2t} + 5e^{-2t} + 10te^{-2t}\right]u(t)$$

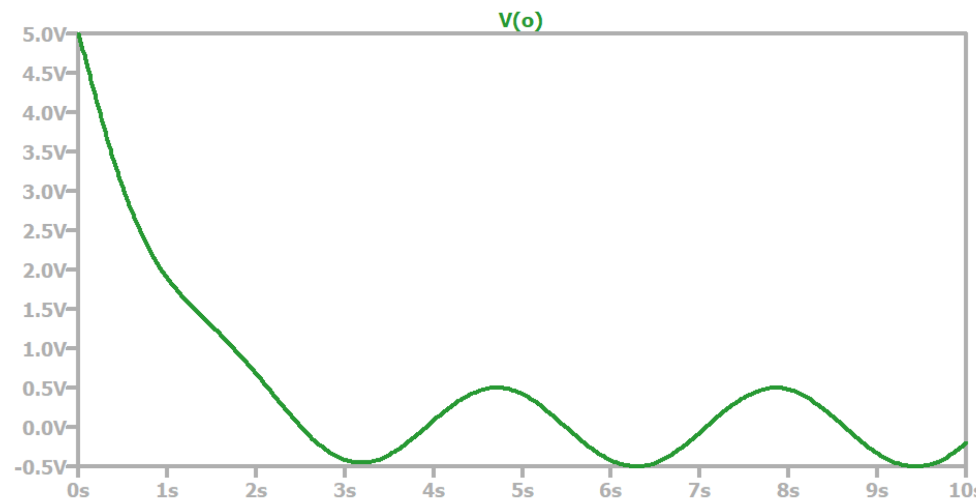
$$\boxed{v_o(t) = \left[5.5e^{-2t} + 7te^{-2t} - \frac{1}{2}\cos(2t)\right]u(t)}$$

Comparison to Simulation

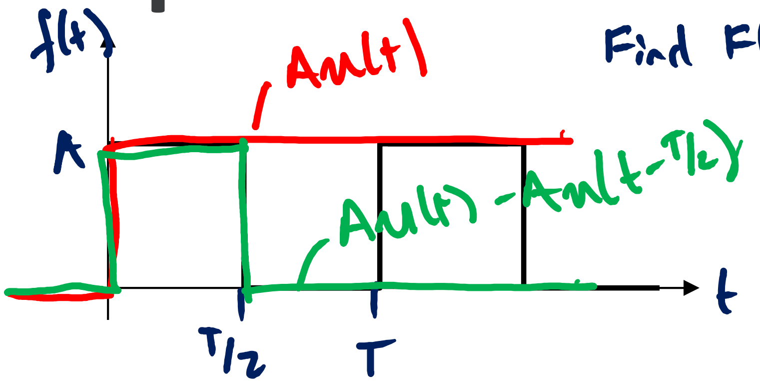








Laplace Transform of Periodic Signals



Find $F(s) = \mathcal{L}\{f(t)\}$

$$f(t) = Au(t) - Au(t-T/2) + Au(t-T) - Au(t-T-T/2) + \cdots$$

$$f(t) = \sum_{n=0}^{\infty} Au(t-nT) - Au\left(t-nT-\frac{T}{2}\right)$$

$$F(s) = A\left[\sum_{n=0}^{\infty} \frac{1}{s}e^{-nTs} - \frac{1}{s}e^{-nTs}e^{-\frac{T}{2}s}\right]$$

$$= A\left[\sum_{n=0}^{\infty} \frac{1}{s}\left(1-e^{-\frac{T}{2}s}\right)e^{-nTs}\right] = A\frac{1}{s}\left(1-e^{-\frac{T}{2}s}\right)\underbrace{\sum_{n=0}^{\infty}\left(e^{-Ts}\right)^{n}}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad |r|<1$$

$$\boxed{F(s) = \frac{A\frac{1}{s}\left(1-e^{-\frac{T}{2}s}\right)}{1-e^{-sT}}} = \frac{F_1(s)}{1-e^{-sT}},$$

$F_1(s)$ is the laplace transform of the first period

exponential s in denominator means repeat every period in time domain