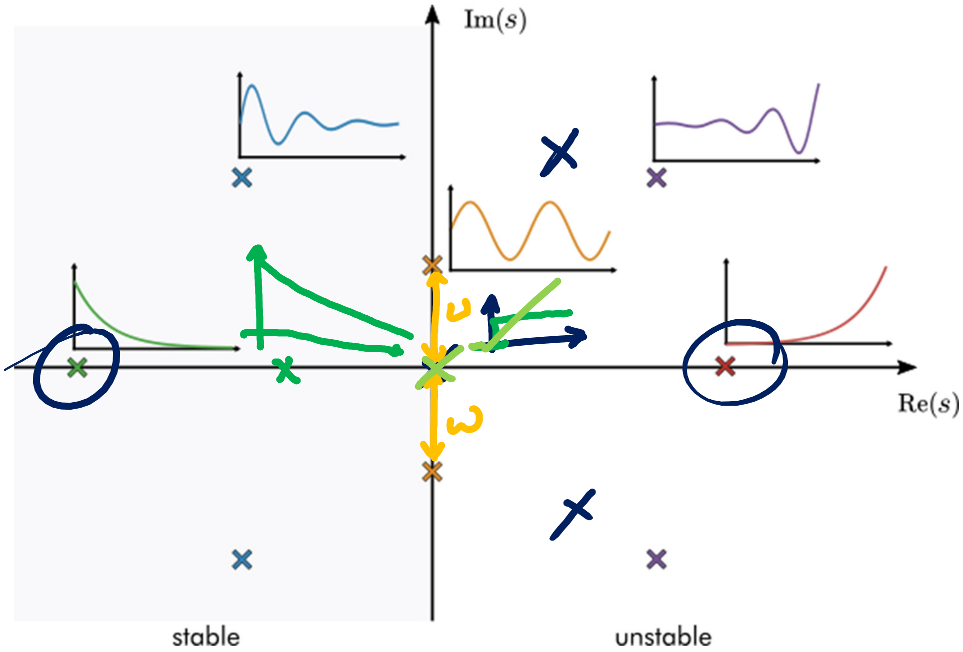
Poles-Zero Plot



Takeaways

- complex poles/zeros always show up in conjugate pairs (required for real-valued time-domain function)

- If all poles in open LHP system is stable / signal is bounded

- If all poles in open LHP jω-axis is in region of convergence (ch 15 - frequency response)

If F(s) has poles at $-\sigma \pm j\omega$

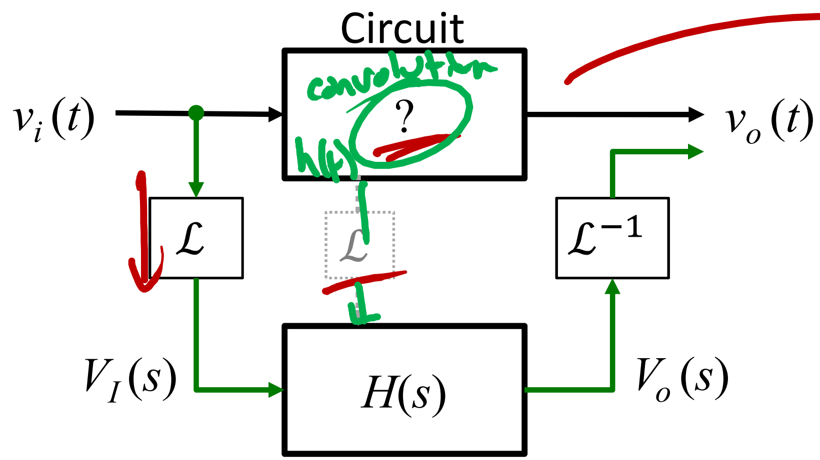
$$F(s) = \frac{N(s)}{\cdots(s-(-\sigma+j\omega))(s-(-\sigma-j\omega))\cdots}$$

$$F(s) = \cdots + \frac{k}{s-(-\sigma+j\omega)} + \frac{k^*}{s-(-\sigma-j\omega)} + \cdots$$

$$f(t) = \cdots + 2|k| e^{-\sigma t} \cos(\omega t - \angle k) + \cdots$$

https://www.mathworks.com/matlabcentral/fileexchange/94635-transfer-function-analysis-of-dynamic-systems

System I/O Relationship



201 approach: solve ODEs

202: $V_I(s) = \mathcal{L}\{v_i(t)\}$

$V_o(s) = H(s)V_I(s)$

$v_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = \mathcal{L}^{-1}\{H(s)V_I(s)\}$

$v_o(t) = \mathcal{L}^{-1}\{H(s)V_I(s)\}$

$= \int_0^\infty h(t-\tau)v_i(\tau)d\tau$ property for laplace transform of a product of two s-domain functions?

$\mathcal{L}^{-1}\{V_I(s)\} = v_i(t)$

$\mathcal{L}^{-1}\{H(s)\} = h(t)$

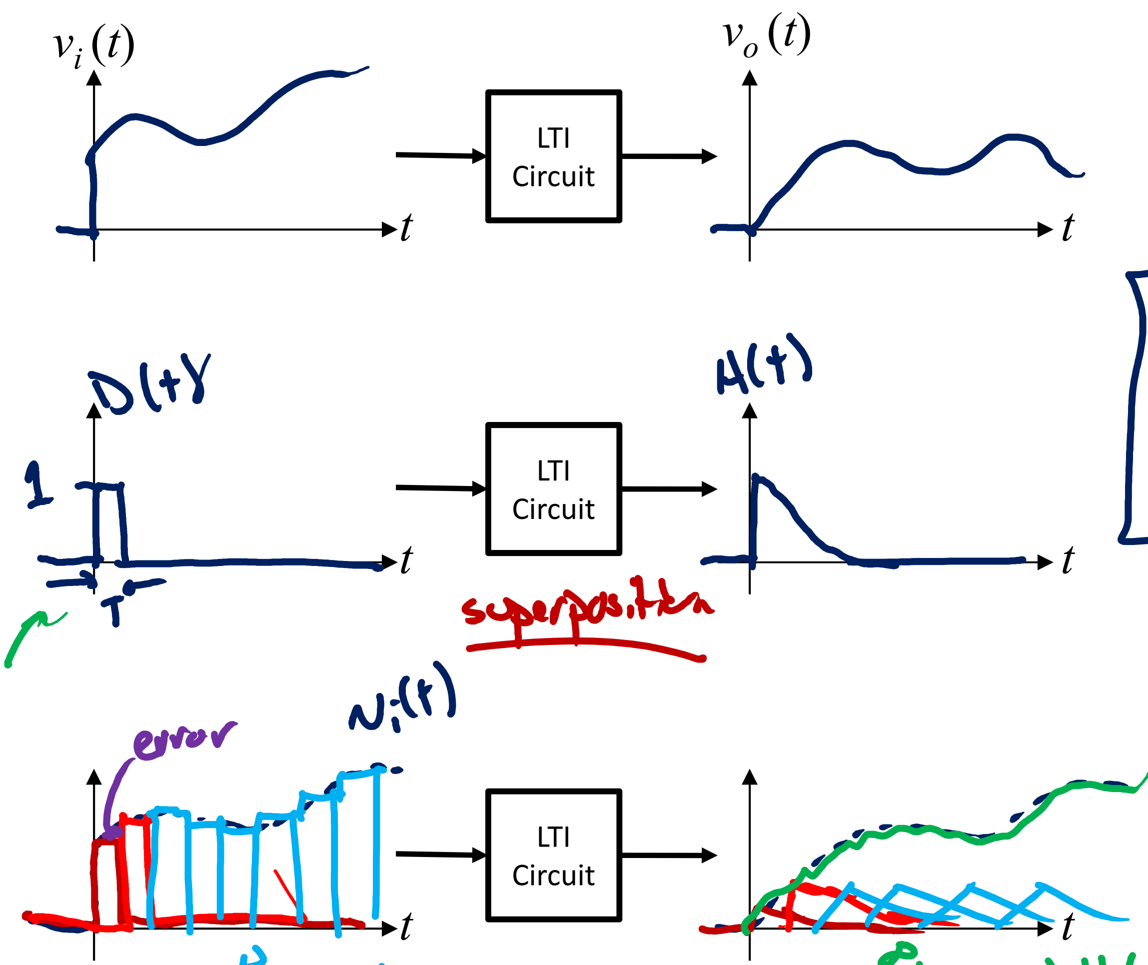
if $V_I(s) = 1 \longleftrightarrow v_i(t) = \delta(t)$

then

$v_o(t) = \mathcal{L}^{-1}\{H(s)\cdot 1\} = \mathcal{L}^{-1}\{H(s)\}$

$\boxed{\mathcal{L}^{-1}\{H(s)\} = h(t) \equiv \text{"impulse response"}}$

Convolution



$v_i(t) \approx \sum_{k=0}^{\infty} v_i(kT)\, D(t-kT)$

$v_o(t) \approx \sum_{k=0}^{\infty} v_i(kT)\, H(t-kT)$

as $T \to 0$

$$v_i(t) = \int_0^{\infty} v_i(\tau)\,\delta(t-\tau)\,d\tau$$

sifting property of $\delta(t)$

$$\boxed{v_o(t) = \int_0^{\infty} v_i(\tau)\,h(t-\tau)\,d\tau = \int_0^{\infty} v_i(t-\tau)\,h(\tau)\,d\tau}$$

Convolution integral

(formally $\int_{-\infty}^{\infty} v_i(\tau)h(t-\tau)\,d\tau$ but with unilateral laplace zero for $t<0$)

The Convolution Integral

Laplace Transform property:

short-hand

$$V_o(s) = V_I(s)\, H(s) \iff v_o(t) = \int_0^\infty h(t-\tau)\, v_i(\tau)\, d\tau = h(t) * v_i(t)$$

$$v_o(t) = \int_0^\infty h(-\tau + t)\, v_i(\tau)\, d\tau$$

flip

shift

at $t=2$

$$v_o(t=2) = \int_0^\infty h(-\tau + 2)\, v_i(\tau)\, d\tau$$