Frequency Response

\[ V_o = \frac{2}{{j\omega}} + \frac{3}{{j\omega + 1}} + \frac{4}{{j\omega + 2}} = \frac{5}{{j\omega + 3}} + 1 \]

\[ \frac{V_o}{V_i} = \frac{1}{{j\omega + 1}} \rightarrow \frac{V_o}{V_i} = H(j\omega) \]

The Frequency Response. It is a complex number with \( \omega \) as the variable. The magnitude and phase shift of our output compared to the input.

\[ H(j\omega) = \frac{1}{j\omega} \times \left( \frac{\omega^2}{j(\omega + 1)} \right) \]

\[ A(j\omega) = \frac{1}{1 + j\omega} \]

Output as:

\[ V_o = V_i H(j\omega) = 1/\omega \left( 0 + j\omega \right) \]

Magnitude

Phase

Graphs:

- Log-log graph of magnitude vs. frequency
- Bode plot of magnitude and phase vs. frequency
Frequency Response and Circuit Behavior
Complex Poles

Bode Diagram

Magnitude (abs)

Frequency (kHz)

Step Response

Amplitude

Time (seconds)

×10^-3
Bode Plots
dB Scale

Decibels

\[ \| G \|_{dB} = 20 \log_{10}(\| G \|) \]

Decibels of quantities having units (impedance example): normalize before taking log

\[ \| Z \|_{dB} = 20 \log_{10}\left( \frac{\| Z \|}{R_{base}} \right) \]

Table 8.1. Expressing magnitudes in decibels

<table>
<thead>
<tr>
<th>Actual magnitude</th>
<th>Magnitude in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>− 6dB</td>
</tr>
<tr>
<td>1</td>
<td>0 dB</td>
</tr>
<tr>
<td>2</td>
<td>6 dB</td>
</tr>
<tr>
<td>5 = 10/2</td>
<td>20 dB − 6 dB = 14 dB</td>
</tr>
<tr>
<td>10</td>
<td>20dB</td>
</tr>
<tr>
<td>1000 = 10³</td>
<td>3 \cdot 20dB = 60 dB</td>
</tr>
</tbody>
</table>

5Ω is equivalent to 14dB with respect to a base impedance of \( R_{base} = 1\Omega \), also known as 14dBΩ.

60dBμA is a current 60dB greater than a base current of 1μA, or 1mA.
Logarithm Review
Plotting on Logarithmic Axes

\[ \|H(j\omega)\|_{dB} \]

\[ H(s) = s \]
Single Pole Response
Magnitude of Single Pole Response
Asymptotic Behavior

\[ \|H(j\omega)\|_{dB} \]

\[ \omega \text{ (log scale)} \]

-80 dB
-60 dB
-40 dB
-20 dB
0 dB
20 dB