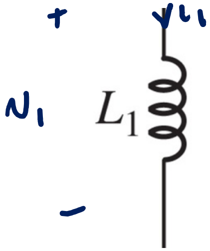
Energy Storage

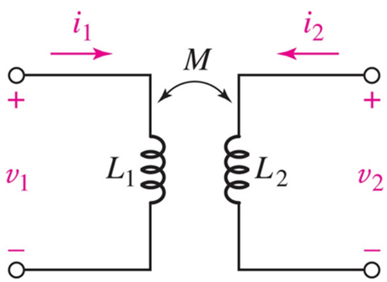


Review

when I_0 current flowing through L_1

$$E_L = \int_0^{t_0} p_L(t)\,dt = \int_0^{t_0} i_1(t)\cdot v_1(t)\,dt = \int_0^{t_0} L\, \underbrace{i_1(t)\frac{di_1}{dt}}\,dt$$

$$= L\int_0^{t_0}\frac{1}{2}\left[\frac{d}{dt}\, i_1(t)^2\right]dt = \boxed{\frac{1}{2} L I_0^2} = E_L$$



At t_0 $i_1(t) = I_{o1}$ & $i_2(t) = I_{o2}$

$$E_{12} = \int_0^t \left(v_1(t)\, i_1(t) + v_2(t)\, i_2(t)\right)dt$$

$$= \int_0^t \left(L_1\, i_1(t)\frac{di_1}{dt} \pm M\, i_1(t)\frac{di_2}{dt} + L_2\, i_2(t)\frac{di_2}{dt} \pm M\, i_2(t)\frac{di_1}{dt}\right)dt$$

$$= \frac{1}{2} L_1 I_{o1}^2 + \frac{1}{2} L_2 I_{o2}^2 \pm \int_0^t M \underbrace{\left(i_1(t)\frac{di_2}{dt} + i_2\frac{di_1}{dt}\right)}_{=\frac{d}{dt}(i_1\cdot i_2)} dt$$

$$\boxed{E_{12} = \frac{1}{2} L_1 I_{o1}^2 + \frac{1}{2} L_2 I_{o2}^2 \pm M I_{o1} I_{o2}}$$

Starting from zero current & bringing up $i_1(t) = I_{01}$ & $i_2(t) = I_{02}$, must be true that

$$E_{12} = \frac{1}{2} L_1 I_{01}^2 + \frac{1}{2} L_2 I_{02}^2 - M I_{01} I_{02} \geq \phi$$

$$M < \frac{\frac{1}{2} L_1 I_{01}^2 + \frac{1}{2} L_2 I_{02}^2}{I_{01} I_{02}} = \frac{1}{2} L_1 \frac{I_{01}}{I_{02}} + \frac{1}{2} L_2 \frac{I_{02}}{I_{01}}$$

Let $x = \frac{I_{01}}{I_{02}}$ $\longrightarrow$ $M < \frac{1}{2} L_1 x + \frac{1}{2} L_2 \frac{1}{x}$

to find minimum:

$$\frac{\partial M}{\partial x} = \frac{1}{2} L_1 + \frac{1}{2} L_2 \frac{-1}{x^2} = \phi \rightarrow x = \sqrt{\frac{L_2}{L_1}}$$

$$\frac{\partial^2 M}{\partial x^2} = \frac{1}{2} L_2 \frac{+2}{x^3}$$

$$M < \frac{1}{2} L_1 \sqrt{\frac{L_2}{L_1}} + \frac{1}{2} L_2 \sqrt{\frac{L_1}{L_2}} = \frac{1}{2}\sqrt{\frac{L_1^2 L_2}{L_1}} + \frac{1}{2}\sqrt{\frac{L_2^2 L_1}{L_2}}$$

$$\boxed{M \leq \sqrt{L_1 L_2}}$$

Coupling Coefficient

Define $k = \frac{M}{\sqrt{L_1 L_2}}$ is the "coupling coefficient"

$0 \le k \le 1$

$k = 0 \rightarrow$ two seperate inductors

$\vdots \quad \}$ a pair of coupled inductors

$k = 1 \rightarrow$ perfect coupling = "transformer"

Transformers

Special case when $k=1 \longleftrightarrow$ perfect coupling $\longleftrightarrow$ $\underline{M=\sqrt{L_1 L_2}}$

$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} \pm M \dfrac{di_2}{dt} \\ v_2 = \pm M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases} \xleftrightarrow{M=\sqrt{L_1 L_2}} \begin{cases} v_1 = L_1 \dfrac{di_1}{dt} \pm \sqrt{L_1 L_2}\dfrac{di_2}{dt} \\ v_2 = \pm \sqrt{L_1 L_2}\dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} \end{cases}$$

$$\downarrow$$

$$v_1 = v_2 \sqrt{\frac{L_1}{L_2}}$$

$$v_1 = v_2 \sqrt{\frac{\cancel{\alpha_1} N_1^2}{\cancel{\alpha_2} N_2^2}} \qquad \alpha_1 = \alpha_2 \text{ if } k=1$$

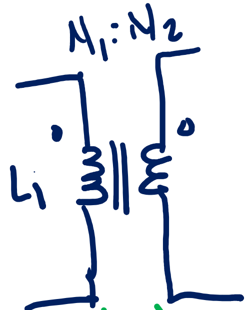
$$\downarrow$$

$$\boxed{v_1 = v_2 \frac{N_1}{N_2}}$$

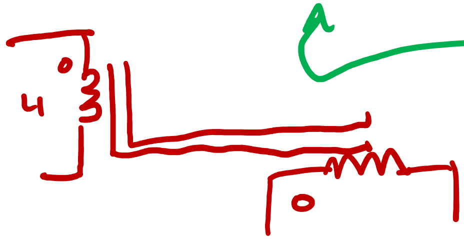
$$\downarrow$$

"turns ratio"

Symbol for transformer



$1:N \longrightarrow N = \dfrac{N_2}{N_1}$



Ideal Transformer

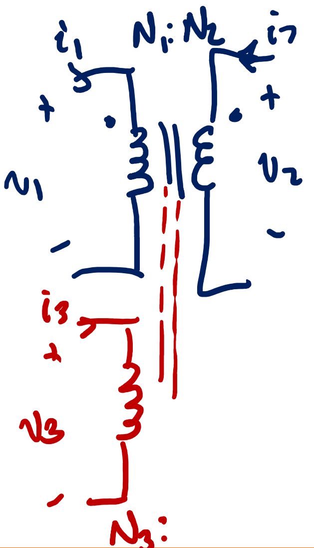
↳ L_1 & L_2 are "large"

↳ $k=1$

Recall: Inductors (& transformers) cannot have DC voltage applied

- $v = L\frac{di}{dt}$ → $i = \frac{1}{L}\int_0^t v\,dt$ so current goes to ∞
- Materials: core materials saturate at high flux causing inductance to drop to nearly zero.
- $v = N\frac{d\Phi}{dt}$ → Need to have time-varying (non-DC signals)

When L_1 & L_2 are "large" i_1 & i_2 are negligible & no energy storage in the transformer



$$\boxed{v_1 = v_2\frac{N_1}{N_2}}$$

$$v_1 i_1 + v_2 i_2 = \emptyset$$

$$v_2\frac{N_1}{N_2} i_1 + v_2 i_2 = \emptyset$$

$$\boxed{N_1 i_1 + N_2 i_2 = \emptyset}$$

$$\boxed{\begin{gathered}\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3} \\ i_1 N_1 + i_2 N_2 + i_3 N_3 = 0\end{gathered}}$$