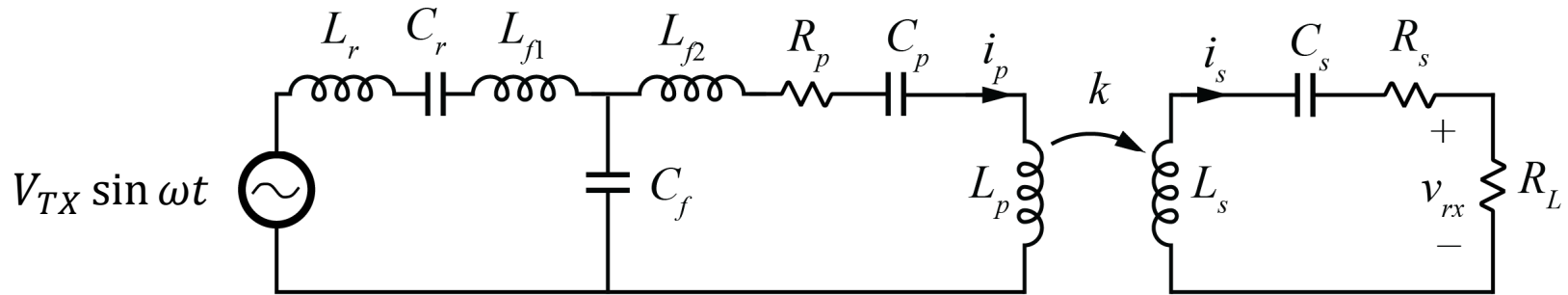
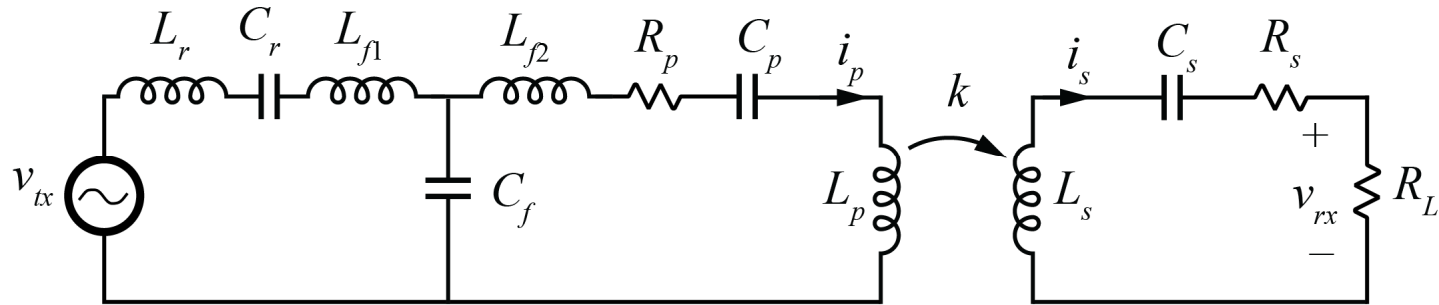


# CHAPTER 10: SINUSOIDAL STEADY-STATE

# Motivation



# Motivation



Numerical

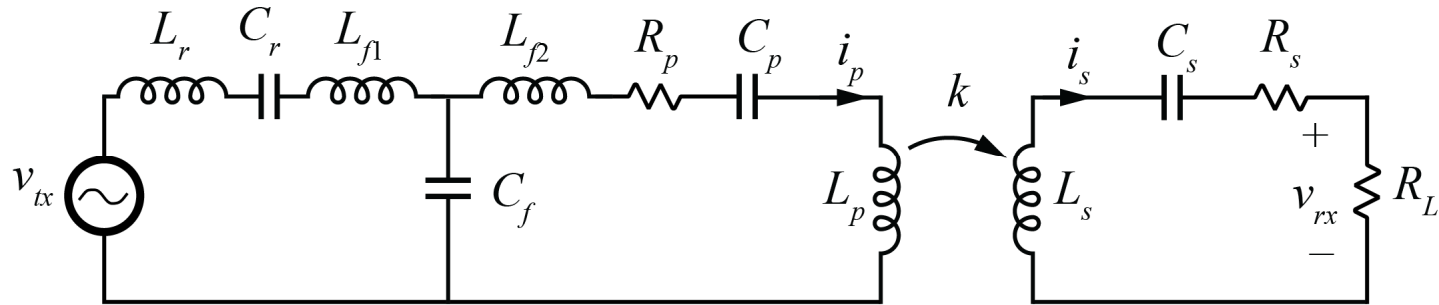
Analytical

Time Domain

Frequency Domain

$$\left\{ \begin{array}{l} \frac{dA}{dt} = \beta_A \cdot \frac{D^n}{K_{DA}^n + D^n} \cdot \left( 1 - \frac{C^n}{K_{CA}^n + C^n} \right) - \alpha_A \cdot A \\ \frac{dX}{dt} = \beta_X \cdot \frac{A^n}{K_{AX}^n + A^n} - \alpha_X \cdot X \\ \frac{dC}{dt} = \beta_C \cdot \frac{(A+X)^n}{K_{AXC}^n + (A+X)^n} - \alpha_C \cdot C \\ \frac{dB}{dt} = \beta_B \cdot \frac{C^n}{K_{CB}^n + C^n} \cdot \left( 1 - \frac{D^n}{K_{DB}^n + D^n} \right) - \alpha_B \cdot B \\ \frac{dY}{dt} = \beta_Y \cdot \frac{B^n}{K_{BY}^n + B^n} - \alpha_Y \cdot Y \\ \frac{dD}{dt} = \beta_D \cdot \frac{(B+Y)^n}{K_{BYD}^n + (B+Y)^n} - \alpha_D \cdot D \end{array} \right.$$

# Motivation

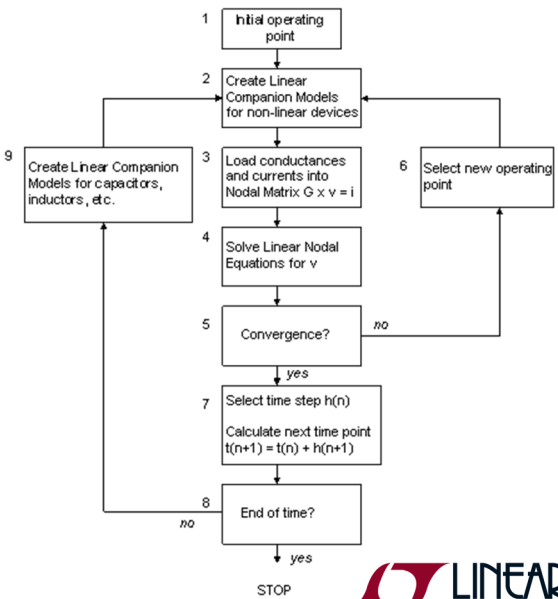


Numerical

Analytical

Time Domain

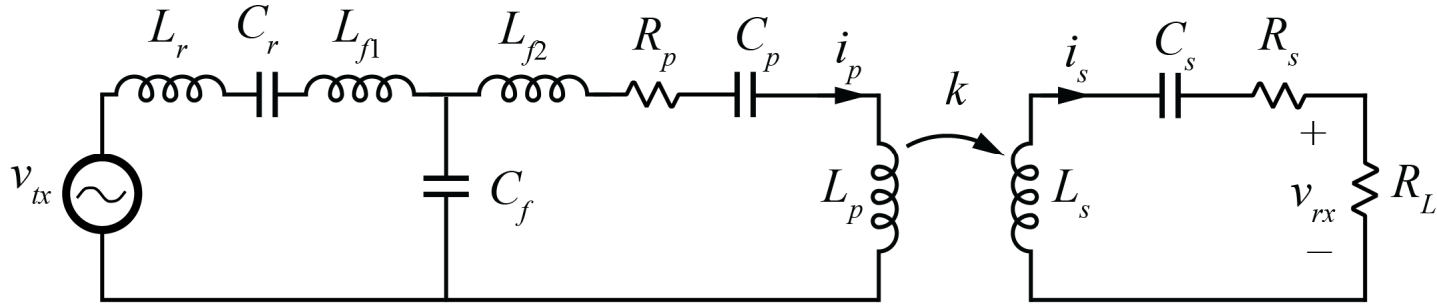
Frequency Domain



$$\left\{ \begin{array}{l} \frac{dA}{dt} = \beta_A \cdot \frac{D^n}{K_{DA}^n + D^n} \cdot \left( 1 - \frac{C^n}{K_{CA}^n + C^n} \right) - \alpha_A \cdot A \\ \frac{dX}{dt} = \beta_X \cdot \frac{A^n}{K_{AX}^n + A^n} - \alpha_X \cdot X \\ \frac{dC}{dt} = \beta_C \cdot \frac{(A+X)^n}{K_{AXC}^n + (A+X)^n} - \alpha_C \cdot C \\ \frac{dB}{dt} = \beta_B \cdot \frac{C^n}{K_{CB}^n + C^n} \cdot \left( 1 - \frac{D^n}{K_{DB}^n + D^n} \right) - \alpha_B \cdot B \\ \frac{dY}{dt} = \beta_Y \cdot \frac{B^n}{K_{BY}^n + B^n} - \alpha_Y \cdot Y \\ \frac{dD}{dt} = \beta_D \cdot \frac{(B+Y)^n}{K_{BYD}^n + (B+Y)^n} - \alpha_D \cdot D \end{array} \right.$$



# Motivation

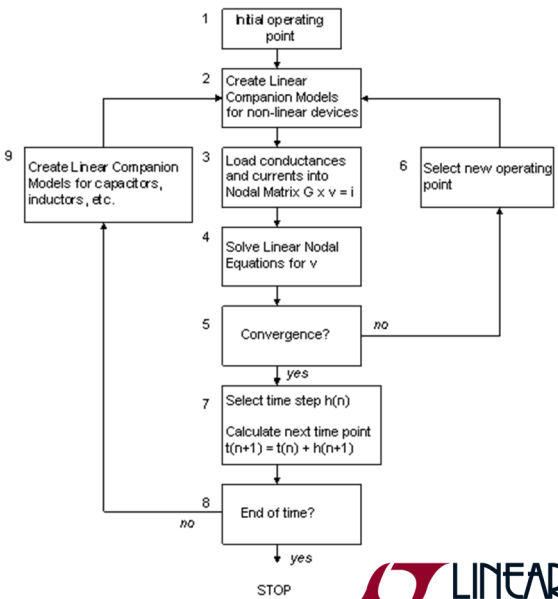


Numerical

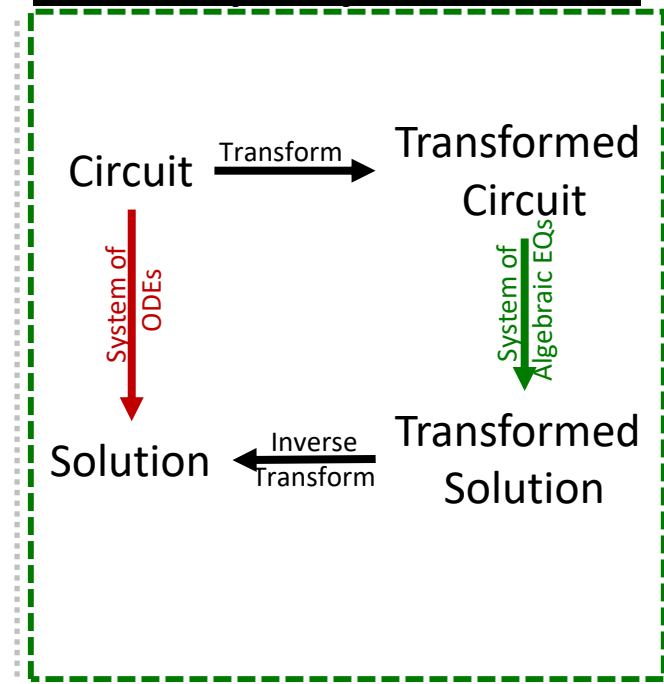
Analytical

Time Domain

Frequency Domain



$$\left\{ \begin{array}{l} \frac{dA}{dt} = \beta_A \cdot \frac{D^n}{K_{DA}^n + D^n} \cdot \left( 1 - \frac{C^n}{K_{CA}^n + C^n} \right) - \alpha_A \cdot A \\ \frac{dX}{dt} = \beta_X \cdot \frac{A^n}{K_{AX}^n + A^n} - \alpha_X \cdot X \\ \frac{dC}{dt} = \beta_C \cdot \frac{(A+X)^n}{K_{AXC}^n + (A+X)^n} - \alpha_C \cdot C \\ \frac{dB}{dt} = \beta_B \cdot \frac{C^n}{K_{CB}^n + C^n} \cdot \left( 1 - \frac{D^n}{K_{DB}^n + D^n} \right) - \alpha_B \cdot B \\ \frac{dY}{dt} = \beta_Y \cdot \frac{B^n}{K_{BY}^n + B^n} - \alpha_Y \cdot Y \\ \frac{dD}{dt} = \beta_D \cdot \frac{(B+Y)^n}{K_{BYD}^n + (B+Y)^n} - \alpha_D \cdot D \end{array} \right.$$



# Form of the Solution

$N^{\text{th}}$  order circuit with sinusoidal input described by

$$k_N \frac{d^N}{dt^N} v_o(t) + \cdots + k_2 \frac{d^2}{dt^2} v_o(t) + k_1 \frac{d}{dt} v_o(t) + k_0 v_o(t) = V_I \sin(\omega t)$$

$$\sum_{i=0}^N k_i \frac{d^i}{dt^i} v_o(t) = V_I \sin(\omega t)$$

solution for  $v_o(t)$  will be of the form

$$v_o(t) = v_{o,h}(t) + v_{o,p}(t)$$

# Transient Response

$v_{o,h}(t)$  is the *homogeneous solution* to the differential equation, the *natural response* of the system, or the *transient response* of the system. For any non-ideal (damped) circuit,  $v_{o,h}(t)$  will tend to zero over time

$v_{o,h}(t)$  is the solution to the equation

$$\sum_{i=0}^N k_i \frac{d^i}{dt^i} v_{o,h}(t) = 0$$

which will be of the form

$$v_{o,h}(t) = \sum_{i=0}^N A_i e^{s_i t}$$

$s_i \rightarrow$  roots of characteristic polynomial

$A_i \rightarrow$  determined by initial conditions

Note: some of the time constants ( $s_i$ ) of a circuit are independent of the input

# Steady-State Response

$v_{o,p}(t)$  is the *particular solution* to the differential equation, the *forced response* of the system, or the *steady-state response* of the system. In general, it does not tend to zero, if non-zero inputs are present.

$v_{o,p}(t)$  is the non-zeroing solution to the equation


$$\sum_{i=0}^N k_i \frac{d^i}{dt^i} v_{o,p}(t) = V_I \sin(\omega t)$$



# LTI Systems

For a function  $f(\cdot)$

## Linearity

$$\text{if: } v_{o,1}(t) = f(v_{i,1}(t)) \quad v_{o,2}(t) = f(v_{i,2}(t))$$


$$\text{then: } av_{o,1}(t) + bv_{o,2}(t) = f(av_{i,1}(t) + bv_{i,2}(t))$$

## Time Invariance

$$\text{if: } v_o(t) = f(v_i(t))$$

$$\text{then: } v_o(t - T) = f(v_i(t - T))$$

$$\text{LTI: } v_x(t) + v_y(t), \quad \alpha v_x(t), \quad \frac{dv_x(t)}{dt}, \quad \int_0^t v_x(t)dt \quad (\text{neglecting ICs})$$

$$\text{Not LTI: } v_x(t) \cdot v_y(t), \quad v_x(t)^2, \quad |v_x(t)|$$

# Preview of Frequency Domain