Announcements

- No Phasor analysis expected in HW2
- HW1 graded
  - 9.56 & 9.57 – review solutions
  - 13.5: $L_1 = 0.5L_2 = 1\ \mathrm{mH}$

    $L_1 = 1mH$

    $L_2 = 2mH$
- Quiz on Wednesday 2/16
  - Covers Chapter 13, HW 1&2, Mutual inductance and transformers

Phasor Notation

$v(t) = A\cos(\omega t + \varphi) = \mathrm{Re}\{A e^{j\omega t} e^{j\varphi}\}$

$\neq$ phasor transform @ ω

$\underline{V} = A e^{j\varphi} \iff A \angle \varphi$

bold in book
underbar in lecture

(short hand notation)

$i(t) = B\sin(\omega t + \theta)$
$= B\cos(\omega t + \theta - 90°)$

phasor transform

$\underline{I} = B e^{j(\theta - \frac{\pi}{2})} \longleftrightarrow B \angle (\theta - \frac{\pi}{2})$

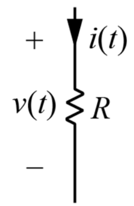
Comments:

- Phasor transform works for volt/current sources & signals
- Everything in the circuit must be at a single frequency ω
- Complex numbers:
  - No imaginary numbers in $v(t)$ or $i(t)$ (in time domain)
  - No time in the phasor domain → no "t" in $\underline{V}$ or $\underline{I}$

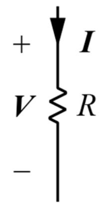
Phasor Circuit Elements

phasor transform

Time Domain | **Phasor Domain**



$v(t) = i(t)R \rightarrow \underline{V} = \underline{I}R$

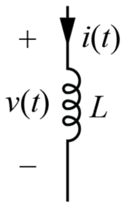


$v(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{V} = Ae^{j\varphi}$

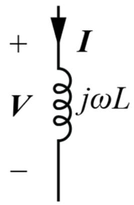
$i(t) = \frac{A}{R}\cos(\omega t + \varphi) \longrightarrow \underline{I} = \frac{A}{R}e^{j\varphi}$

$v(t) = i(t)R$

$$\boxed{\underline{V} = \underline{I}\underline{R}}$$



$v(t) = L\dfrac{di}{dt} \rightarrow \underline{V} = Lj\omega\,\underline{I}$



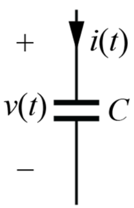
$i(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{I} = Ae^{j\varphi}$

$v(t) = LA\omega\cos(\omega t + \varphi + 90^\circ) \longrightarrow \underline{V} = LA\omega e^{j(\varphi + \frac{\pi}{2})}$

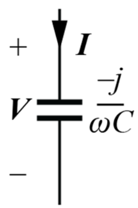
$v(t) = L\dfrac{di}{dt}$

$\underline{V} = j\omega L\,\underbrace{Ae^{j\varphi}}$

$$\boxed{\underline{V} = \underline{j\omega L}\,\underline{I}}$$



$i(t) = C\dfrac{dv}{dt} \rightarrow \underline{I} = Cj\omega\,\underline{V}$



$v(t) = A\cos(\omega t + \varphi) \longrightarrow \underline{V} = Ae^{j\varphi}$

$i(t) = CA\omega\cos(\omega t + \varphi + 90^\circ) \longrightarrow \underline{I} = CA\omega e^{j(\varphi + \frac{\pi}{2})}$

$i(t) = C\dfrac{dv}{dt}$

$\underline{I} = j\omega C\,\underbrace{Ae^{j\varphi}}$

$$\boxed{\underline{V} = \frac{-j}{\underline{\omega C}}\underline{I}} = \boxed{\underline{V} = \frac{1}{\underline{j\omega C}}\underline{I}}$$

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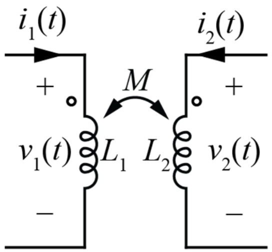
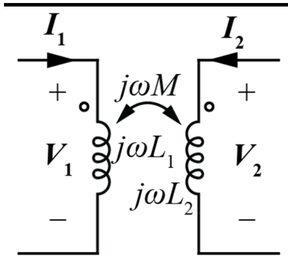
Time Domain

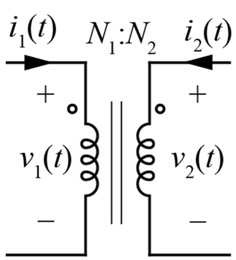
Phasor Domain

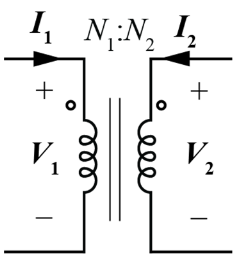




$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \longrightarrow \underline{V_1} = j\omega L_1 \underline{I_1} + j\omega M \underline{I_2}$$

$$v_2(t) = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \longrightarrow \underline{V_2} = j\omega M \underline{I_1} + j\omega L_2 \underline{I_2}$$





$$\frac{v_1(t)}{N_1} = \frac{v_2(t)}{N_2} \longrightarrow \frac{\underline{V_1}}{N_1} = \frac{\underline{V_2}}{N_2}$$

$$N_1 i_1(t) + N_2 i_2(t) = 0 \longrightarrow N_1 \underline{I_1} + N_2 \underline{I_2} = \emptyset$$

Impedance

Phasor equivalent of ohm's law

$$\underline{V} = \underline{I}\, Z$$

Z = "Impedance" = $R + jX$

$\mathrm{Re}\{Z\} = R$ "Resistance"

$\mathrm{Im}\{Z\} = X$ "Reactance"

$$\begin{cases} Z_R = R, & \text{Resistor} \\ Z_L = j\omega L, & \text{Inductor} \\ Z_C = \dfrac{-j}{\omega C}, & \text{capacitor} \end{cases}$$

All have units of ohms (Z, R, X)

Y = "Admittance" = $\dfrac{1}{Z}$ = $G + jB$

conductance

susceptance

→ All units of Siemens

$$Y = \frac{1}{Z} = \frac{1}{R+jX} \neq \cancel{\frac{1}{R} + j\frac{1}{X}}$$ No.

Yes! ↳ $$\frac{1}{R+jX}\,\frac{(R-jX)}{(R-jX)} = \frac{R-jX}{R^2+X^2} = \frac{R}{R^2+X^2} - j\,\frac{X}{R^2+X^2}$$

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