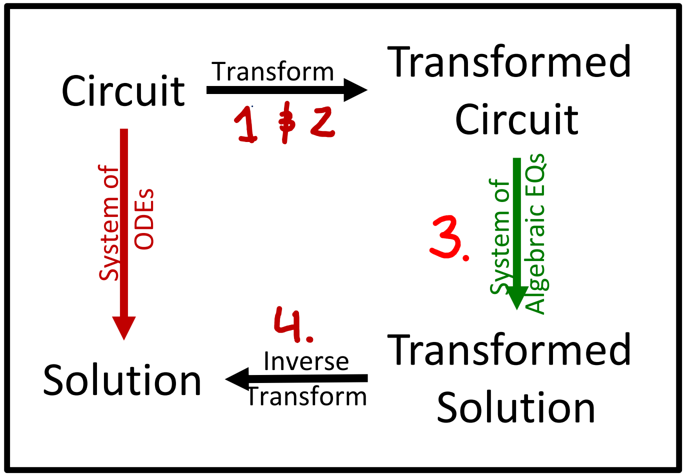
Phasor Circuit Analysis



Start: LTI circuit with single-frequency sinusoidal source(s) $ want to find steady-state solution

1: Transform all sources $ signals into their phasor equivalents

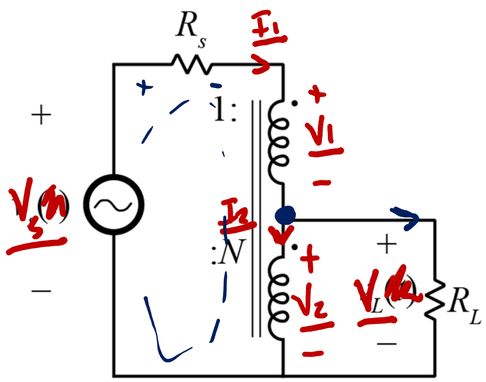
2: Transform all passives into their impedances

3: Solve the circuit
- Can use all 201 analysis techniques for DC, resistor-only circuits

4: Transform phasor voltage or current back into the time domain

Example Problem

$\omega = 2\pi 60$



Find $v_L(t)$ for $v_s(t) = 170\cos(2\pi 60 t)$ and for $R_s = 10\ \Omega$, $N = 0.1$, and $R_L = 50\ \Omega$

No differential equations, so no benefit to phasors

1. $v_s(t) \longrightarrow \underline{V_s} = 170 \angle 0° \text{ or } 170 e^{j0} = 170$

   $v_L(t) \longrightarrow \underline{V_L} = V_A \angle \varphi$

2. $R_s \to R_s \qquad R_L \to R_L$

$$\frac{\underline{V_1}}{1} = \frac{\underline{V_2}}{N} \qquad \& \qquad \underline{I_1} + N\underline{I_2} = \emptyset \longrightarrow \underline{I_2} = -\frac{1}{N}\underline{I_1}$$

3. loop

$$\underline{V_s} = \underline{V_1} + \underline{V_2} + \underline{I_1} R_s$$

$$\underline{V_s} = \left(\frac{1}{N} + 1\right)\underline{V_L} + \underline{I_1} R_s$$

node

$$\underline{I_1} = \underline{I_2} + \frac{\underline{V_L}}{R_L}$$

$$\underline{I_1}\left(1 + \frac{1}{N}\right) = \frac{\underline{V_L}}{R_L}$$

$$\underline{V_s} = \left(\frac{1}{N} + 1\right)\underline{V_L} + R_s \frac{\underline{V_L}}{R_L} \frac{1}{1 + \frac{1}{N}}$$

$$\underline{V_s} = \underline{V_L}\left[\frac{N+1}{N} + \frac{R_s}{R_L}\,\frac{N}{N+1}\right]$$

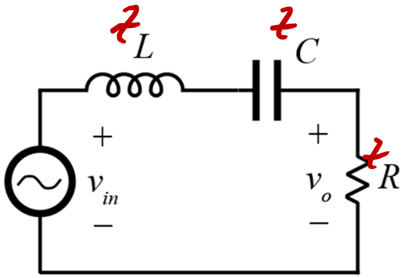
$$\boxed{\underline{V_L} = \underline{V_s}\,\frac{1}{\frac{N+1}{N} + \frac{R_s}{R_L}\,\frac{N}{N+1}}}$$

plug in all #'s : $\underline{V_L} = 15.5\,e^{j0} = 15.5$

4 $\boxed{v_L(t) = 15.5\cos(2\pi 60\,t + 0^\circ)}$

FYI: if $R_s \to 0$ $\quad V_L = \underline{V_s}\,\frac{N}{N+1}$

Resonance Example



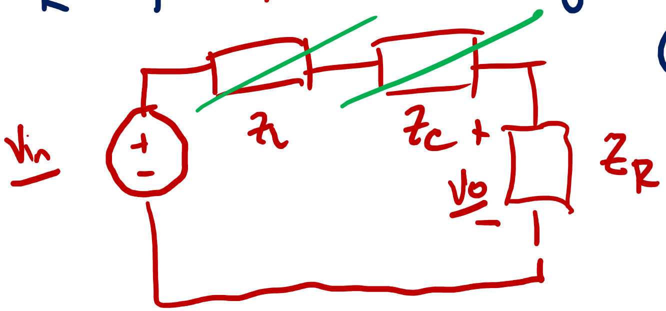
$= 10\cos(\omega t - 90°)$

Find $v_o(t)$ for $v_{in}(t) = 10\sin(\omega t)$ and $\omega = \underline{2\pi 100\ \text{kHz}}$, $R = 10\ \Omega$, $L = 10\ \mu\text{H}$, and $C = 253$ nF

1. $v_{in}(t) \rightarrow \underline{V_{in}} = 10 \angle -90°$ or $10e^{j\frac{-\pi}{2}}$

   $v_o(t) \rightarrow \underline{V_o}$

2. $R \rightarrow R = Z_R$ $\quad L \rightarrow j\omega L = Z_L$ $\quad C \rightarrow \frac{-j}{\omega C} = Z_C$



3. $$\boxed{\underline{V_o} = \underline{V_{in}} \frac{Z_R}{Z_R + Z_L + Z_C}}$$

$R = R = 10\Omega$

$Z_L = j\omega L = j(2\pi\, 100e3)(10e{-6})$

$= j(2\pi)$

$Z_C = \frac{-j}{\omega C} = \frac{-j}{(2\pi\, 100e3)(253e{-9})} = -j(2\pi)$

4. $\underline{V_o} = 10e^{j\frac{-\pi}{2}}$

   $v_o(t) = 10\cos(\omega t - 90°)$

   $\boxed{= 10\sin(\omega t)}$

Reactance and Resonance

Back at the end of 201

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \text{resonant frequency}$$

Now:

$$Z_L = j\omega L \qquad Z_C = \frac{-j}{\omega C}$$

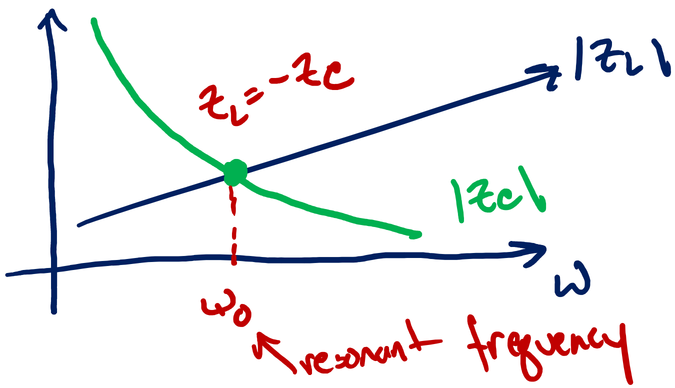
$$Z_L = -Z_C$$

$$j\omega L = -\left(\frac{-j}{\omega C}\right)$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$\lim_{\omega \to 0} \approx DC$$

as $\omega \to 0 \approx DC$

$Z_C \to \infty$ cap $\to$ open

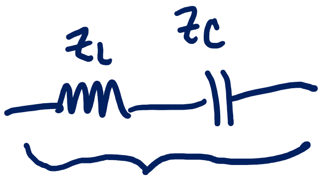
$Z_L \to 0$ inductor $\to$ short

as $\omega \to \infty$

$Z_C \to 0$ cap $\to$ short

$Z_L \to \infty$ inductor $\to$ open

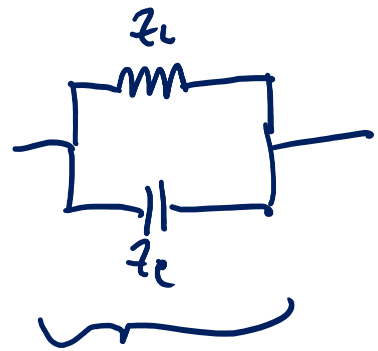
@ $\omega_0 = \frac{1}{\sqrt{LC}}$ $\qquad Z_L = -Z_C$



$Z_{eq} = Z_L + Z_C$

$= \emptyset$ @ resonance

(short)



$$Z_{eq} = \frac{Z_L Z_C}{\underbrace{Z_L + Z_C}} \to \emptyset$$

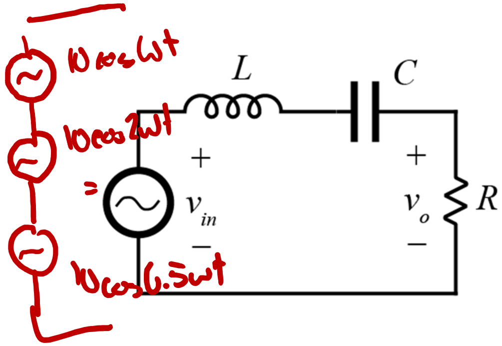
$Z_{eq} \to \infty$ (open)

@ resonance

Phasor Superposition

3 frequencies

phasor analysis doesn't apply



Find $v_o(t)$ for $v_{in}(t) = 10\cos(\omega t) + 10\cos(2\omega t) + 10\cos(0.5\omega t)$ and $\omega = 2\pi 100$ kHz, $R = 10\ \Omega$, $L = 10\ \mu$H, and $C = 253$ nF