1. Use the Myhill-Nerode theorem to prove whether or not each language is regular. If the
language is regular, you should list each canonical equivalence class. If the language is
not regular, you should prove that the set of canonical equivalence classes is infinite.
For all problems let $\Sigma = \{0, 1\}$.

a. $\{0^i2 \mid i \geq 0\}$

b. $\{1^i0^j \mid i \text{ is divisible by three, } j \text{ is odd}\}$

c. $\{w \mid w \text{ has an equal number of 01 and 10 substrings}\}$

d. $\{1^i0^i1^j \mid i, j \geq 1\}$

2. Prove each statement using closure properties of regular sets, or give a counterexample.

a. Regular sets are closed under infinite union.

b. If $L$ has a non-regular proper subset, then $L$ is non-regular.

c. Every infinite regular set has infinitely many infinite regular proper subsets.

Example: Proof, using the Myhill-Nerode theorem, that the following language is not regular:

$\{ww^R \mid w \in (0 + 1)^*, w^R \text{ is the reverse of } w\}$

Call the above language $L$. Let $S$ be the set of strings $\{0^i1 \mid i \geq 0\}$. Consider any two
distinct strings $x$ and $y$ from set $S$. Let $z = x^R$. Note that $xz \notin L$, but $yz \notin L$, because it
has a different number of 0’s after a pair of 1’s than before it. Therefore, every string in $S$ is in a
different equivalence class of $R_L$. Because $S$ is an infinite set, $L$ is not regular by the
Myhill-Nerode theorem.

(Note that for this particular language, it was easier to show only that some subset of $R_L$’s
equivalence classes are infinite, which is good enough. Also, note that the set $S$ is not
actually in $L$, which makes no difference.)