1. What language does the following CFG generate?

   \[ S \rightarrow \text{aaSb} \mid \text{abSa} \mid \text{baSa} \mid SS \mid \lambda \]

2. Give a CFG generating each language.

   a. \{ a^i b^j \mid i < j \leq 2i \}

   b. \{ a^i b^{i+j} c^j \mid i + j \geq 1 \}

   c. \{ s \mid s \in (a + b)^*, \text{ every a is balanced by a b that comes after it} \}

      (Hint: like parentheses in a math expression)

   d. \{ s \mid s \in (a + b)^*, s \text{ contains equal numbers of a's and b's} \}

   e. \{ a^i b^j c^k \mid i = j \text{ or } j = k \}

3. Simplify the following grammar by applying, in the correct order, the four lemmas we have learned. Show the result after applying each lemma.

   \[
   \begin{align*}
   S &\rightarrow BD \mid A \\
   A &\rightarrow FG \\
   B &\rightarrow aBb \mid C \mid \lambda \\
   C &\rightarrow B \mid \lambda \\
   D &\rightarrow Dc \mid \lambda \\
   E &\rightarrow bBa \mid cD \mid \lambda \\
   F &\rightarrow AG \\
   G &\rightarrow FA \\
   \end{align*}
   \]

4. Show whether or not the following CFG is ambiguous.

   \[ S \rightarrow aS \mid aSbS \mid \lambda \]