1. Prove or disprove: If $I$ is a maximum independent set in $G$, then there exists a minimum coloring of $G$ in which all vertices of $I$ have the same color.

2. We talked a bit in class about greedy coloring. A greedy coloring considers all vertices of a graph in some given order. It assigns color #1 to the first vertex considered, and then for each successive vertex, it assigns the lowest numbered color that maintains a valid coloring.

The Southeastern Conference consists of universities from Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, Missouri, South Carolina, Tennessee and Texas. Let $G_{SEC}$ be a graph in which vertices represent states and edges represent borders between states.

a. Draw $G_{SEC}$.

b. Determine the chromatic number of $G_{SEC}$.

c. Give an ordering of states for which greedy coloring of $G_{SEC}$ results in the minimum number of colors.

d. Give an ordering of states for which greedy coloring of $G_{SEC}$ results in more than the minimum number of colors.

3. Problem 8.1.2 (rephrased for clarity): Show that if each pair of odd cycles in a graph $G$ has at least one vertex in common, then $\chi \leq 5$.

4. Problem 8.1.10(a): Show that $\chi(G_1 \lor G_2) = \chi(G_1) + \chi(G_2)$. (See the book’s glossary for definitions.)

5. Problem 8.1.10(b): Show that $G_1 \lor G_2$ is color critical if and only if both $G_1$ and $G_2$ are color critical.