

B. Pattern Formation

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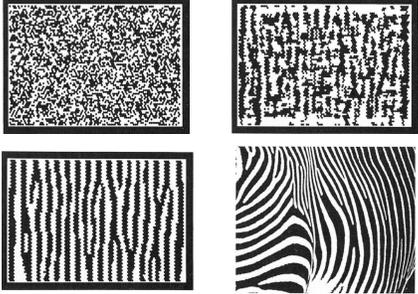
Differentiation & Pattern Formation



- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication

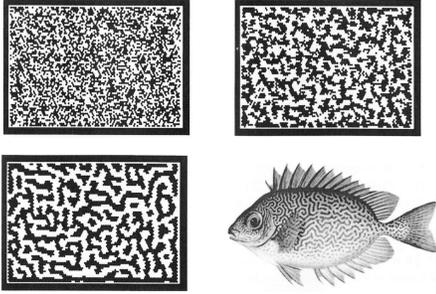
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Zebra



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figs. from Camazine & al.: *Self-Org. Biol. Sys.*

Vermiculated Rabbit Fish



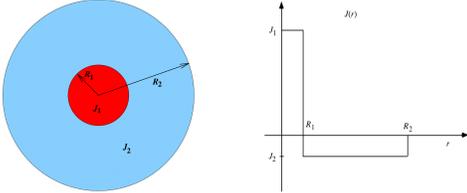
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figs. from Camazine & al.: *Self-Org. Biol. Sys.*

Activation & Inhibition in Pattern Formation

- Color patterns typically have a characteristic length scale
- Independent of cell size and animal size
- Achieved by:
 - short-range activation \Rightarrow local uniformity
 - long-range inhibition \Rightarrow separation

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Interaction Parameters



- R_1 and R_2 are the interaction ranges
- J_1 and J_2 are the interaction strengths

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CA Activation/Inhibition Model

- Let states $s_i \in \{-1, +1\}$
- and h be a bias parameter
- and r_{ij} be the distance between cells i and j
- Then the state update rule is:

$$s_i(t+1) = \text{sign} \left[h + J_1 \sum_{r_{ij} < R_1} s_j(t) + J_2 \sum_{R_1 \leq r_{ij} < R_2} s_j(t) \right]$$

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Example

$(R_1=1, R_2=6, J_1=1, J_2=-0.1, h=0)$

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figs. from Bar-Yam

Effect of Bias

$(h = -6, -3, -1; 1, 3, 6)$

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figs. from Bar-Yam

Effect of Interaction Ranges

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figs. from Bar-Yam

Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation: Fur

[RunAICA.nlogo](#)

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Differential Interaction Ranges

- How can a system using strictly local interactions discriminate between states at long and short range?
- E.g. cells in developing organism
- Can use two different *morphogens* diffusing at two different rates
 - activator diffuses slowly (short range)
 - inhibitor diffuses rapidly (long range)

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Digression on Diffusion

- Simple 2-D diffusion equation:

$$\dot{A}(x,y) = c\nabla^2 A(x,y)$$
- Recall the 2-D Laplacian:

$$\nabla^2 A(x,y) = \frac{\partial^2 A(x,y)}{\partial x^2} + \frac{\partial^2 A(x,y)}{\partial y^2}$$
- The Laplacian (like 2nd derivative) is:
 - positive in a local minimum
 - negative in a local maximum

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Reaction-Diffusion System

diffusion

$$\frac{\partial A}{\partial t} = d_A \nabla^2 A + f_A(A,I)$$

+

$$\frac{\partial I}{\partial t} = d_I \nabla^2 I + f_I(A,I)$$

reaction

$$\frac{\partial}{\partial t} \begin{pmatrix} A \\ I \end{pmatrix} = \begin{pmatrix} d_A & 0 \\ 0 & d_I \end{pmatrix} \nabla^2 \begin{pmatrix} A \\ I \end{pmatrix} + \begin{pmatrix} f_A(A,I) \\ f_I(A,I) \end{pmatrix}$$

$$\dot{\mathbf{c}} = \mathbf{D}\nabla^2 \mathbf{c} + \mathbf{f}(\mathbf{c}), \text{ where } \mathbf{c} = \begin{pmatrix} A \\ I \end{pmatrix}$$

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Continuous-time Activator-Inhibitor System

- Activator *A* and inhibitor *I* may diffuse at different rates in *x* and *y* directions
- Cell becomes more active if activator + bias exceeds inhibitor
- Otherwise, less active

$$\frac{\partial A}{\partial t} = d_{Ax} \frac{\partial^2 A}{\partial x^2} + d_{Ay} \frac{\partial^2 A}{\partial y^2} + k_A(A + B - I)$$

$$\frac{\partial I}{\partial t} = d_{Ix} \frac{\partial^2 I}{\partial x^2} + d_{Iy} \frac{\partial^2 I}{\partial y^2} + k_I(A + B - I)$$

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NetLogo Simulation of Reaction-Diffusion System

- Diffuse activator in X and Y directions
- Diffuse inhibitor in X and Y directions
- Each patch performs:
 - stimulation = bias + activator – inhibitor + noise
 - if stimulation > 0 then
 - set activator and inhibitor to 100
 - else
 - set activator and inhibitor to 0

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Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation

[Run Pattern.nlogo](#)

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Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation with Continuous State Change

[Run Activator-Inhibitor.nlogo](#)

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Turing Patterns

- Alan Turing studied the mathematics of reaction-diffusion systems
- Turing, A. (1952). The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society B* **237**: 37–72.
- The resulting patterns are known as *Turing patterns*

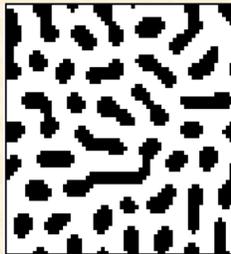
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Abstract Activation/Inhibition Spaces

- Consider two axes of cultural preference
 - E.g. hair length & interpersonal distance
 - Fictitious example!
- Suppose there are no objective reasons for preferences
- Suppose people approve/encourage those with similar preferences
- Suppose people disapprove/discourage those with different preferences
- What is the result?

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Emergent Regions of Acceptable Variation



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A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
 - Positive feedback creates
 - Negative feedback shapes

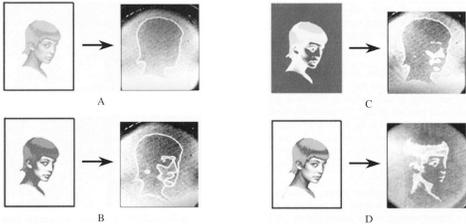
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Reaction-Diffusion Computing

- Has been used for image processing
 - diffusion \Rightarrow noise filtering
 - reaction \Rightarrow contrast enhancement
- Depending on parameters, RD computing can:
 - restore broken contours
 - detect edges
 - improve contrast

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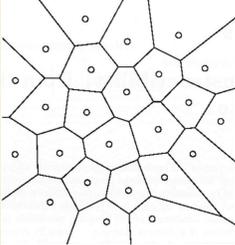
Image Processing in BZ Medium



- (A) boundary detection, (B) contour enhancement, (C) shape enhancement, (D) feature enhancement

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Image < Adamatzky, *Comp. in Nonlinear Media & Autom. Coll.*

Voronoi Diagrams



- Given a set of generating points:
- Construct a polygon around each generating point of set, so all points in a polygon are closer to its generating point than to any other generating points.

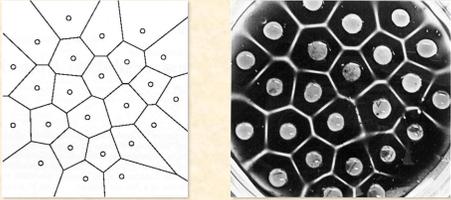
9/8/10 Image < Adamatzky & al., *Reaction-Diffusion Computers* 25

Some Uses of Voronoi Diagrams

- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

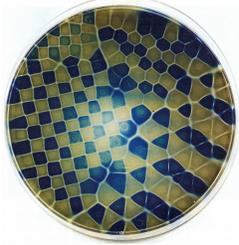
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Computation of Voronoi Diagram by Reaction-Diffusion Processor



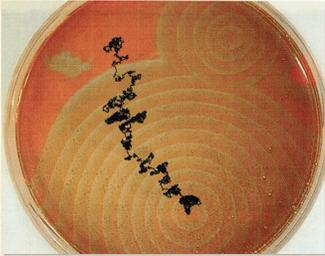
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Mixed Cell Voronoi Diagram



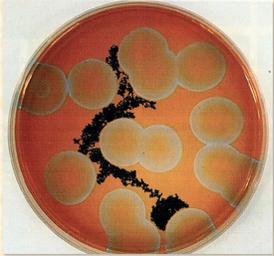
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Path Planning via BZ medium: No Obstacles



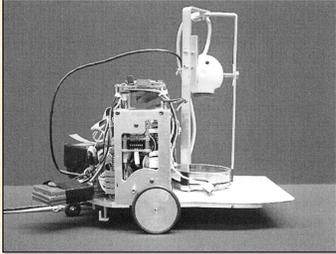
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Path Planning via BZ medium: Circular Obstacles



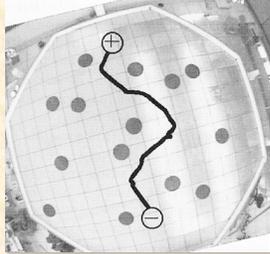
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Mobile Robot with Onboard Chemical Reactor



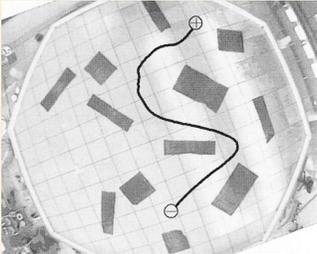
9/8/10 Image < Adamatzky & al., *Reaction-Diffusion Computers* 31

Actual Path: Pd Processor



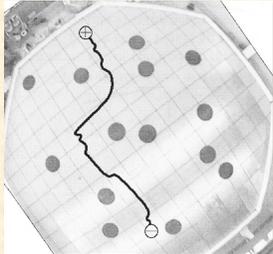
9/8/10 Image < Adamatzky & al., *Reaction-Diffusion Computers* 32

Actual Path: Pd Processor



9/8/10 Image < Adamatzky & al., *Reaction-Diffusion Computers* 33

Actual Path: BZ Processor



9/8/10 Image < Adamatzky & al., *Reaction-Diffusion Computers* 34

Bibliography for Reaction-Diffusion Computing

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Segmentation

(in embryological development)

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Vertebrae

- Humans: 33, chickens: 55, mice: 65, corn snake: 315
- Characteristic of species
- How does an embryo “count” them?
- “Clock and wavefront model” of Cooke & Zeeman (1976).

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Diagram illustrating the development of the neural tube and somites. Labels include: Head, Neural tube, Head mesoderm, Otic vesicle, Somites, Somite formation, Rostrocaudal patterning, Segmental determination, PSM (Presomitic Mesoderm), Tail bud, Tail, and Axis elongation. Source: Nature Reviews | Genetics.

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Diagram illustrating the clock and wavefront model. Part (a) shows a 3D representation of a wavefront moving through a tissue, with a clock mechanism (oscillator) at the rear. Part (b) shows a series of U-shaped tubes representing the progression of an expression wave, with labels for genes like *Sh1*, *Sh2*, *Sh3*, *Sh4*, *Sh5*, *Sh6*, *Sh7*, *Sh8*, *Sh9*, and *Sh10*. Source: Nature Reviews | Genetics.

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Detailed signaling pathway diagram for the clock and wavefront model. It shows the interaction of various signaling molecules and transcription factors. Key components include: FGF (FGFR1, FGFR2, FGFR3), Ras, Raf, MEK, ERK, DUSP6, Sprouty2, Notch1, Dll1, Frizzled (VAN), Wnt4/3, Dkk1, Dsh1, Axin, GSK3, APC, β-catenin, Nkd1, Lfngs, Noto, Noto1, Noto2, Noto3, Noto4, Noto5, Noto6, Noto7, Noto8, Noto9, Noto10, Noto11, Noto12, Noto13, Noto14, Noto15, Noto16, Noto17, Noto18, Noto19, Noto20, Noto21, Noto22, Noto23, Noto24, Noto25, Noto26, Noto27, Noto28, Noto29, Noto30, Noto31, Noto32, Noto33, Noto34, Noto35, Noto36, Noto37, Noto38, Noto39, Noto40, Noto41, Noto42, Noto43, Noto44, Noto45, Noto46, Noto47, Noto48, Noto49, Noto50, Noto51, Noto52, Noto53, Noto54, Noto55, Noto56, Noto57, Noto58, Noto59, Noto60, Noto61, Noto62, Noto63, Noto64, Noto65, Noto66, Noto67, Noto68, Noto69, Noto70, Noto71, Noto72, Noto73, Noto74, Noto75, Noto76, Noto77, Noto78, Noto79, Noto80, Noto81, Noto82, Noto83, Noto84, Noto85, Noto86, Noto87, Noto88, Noto89, Noto90, Noto91, Noto92, Noto93, Noto94, Noto95, Noto96, Noto97, Noto98, Noto99, Noto100. Source: Nature Reviews | Genetics.

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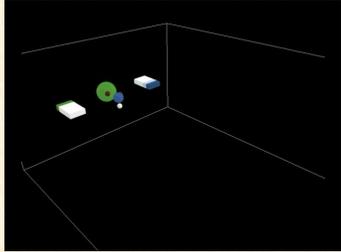
Diagram illustrating the clock and wavefront model over time. It shows a series of U-shaped tubes representing the progression of an expression wave. Labels include: Retinoic acid, FGF/Wnt, T=0, T=1 cycle, T=2 cycles, Somitogenesis, Determination front, and Axis extension. Source: Nature Reviews | Genetics.

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Diagram illustrating the clock and wavefront model showing the bistability window and the dominance of RA and FGF/Wnt. Labels include: RA dominates, FGF/Wnt dominates, Anterior, Posterior, and Bistability window. Source: Nature Reviews | Genetics.

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Simulated Segmentation by Clock-and-Wavefront Process



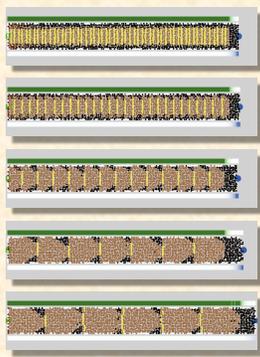
9/8/10 [Run Segmentation-cells-3D.nlogo](#) 43

2D Simulation of Clock-and-Wavefront Process



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Effect of Growth Rate



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NetLogo Simulation of Segmentation

[Run Segmentation.nlogo](#)

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Segmentation References

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6. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology*. Basic Books, 2000.

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