Homework 1 — CS 594 — Due: Sept. 29, 2003

General Instructions

- Please work on this assignment independently.
- You can hand in your assignment electronically (email to zhao@cs.utk.edu) or as hardcopy (put it in Junlong Zhao's mailbox).

Introduction

In class (Lecture 6) we studied an activation-inhibition pattern generation system defined by a CA with the update rule:

$$s_i(t+1) = \operatorname{sign}\left[h + J_1 \sum_{r_{ij}<R_1} s_j(t) + J_2 \sum_{R_1<r_{ij}<R} s_j(t)\right].$$

Recall that r_{ij} represent the distance between cells i and j, so the first summation is over all cells within a distance of R_1 to cell i, and the second summation is over all cells with a distance between R_1 and R_2. The state of a CA can be updated either *synchronously* or *asynchronously*. With synchronous updating, which is what we usually do, all the states are updated simultaneously. With asynchronous updating the cells are updated one at a time (usually in some random order).

This homework assignment explores the stability of this activation-inhibition system; that is, does it inevitably reach a stable state?

Problems

Problem 1

Prove that if the states are updated asynchronously, then the CA must reach a stable state.

Hint: Define the following function (called an *energy* or *Lyapunov* function) of the total state of a CA:

$$E\{\mathbf{s}(t)\} = -\tfrac{1}{2}\sum_{i} s_i(t)\operatorname{sign}\left[h + J_1 \sum_{r_{ij}<R_1} s_j(t) + J_2 \sum_{R_1<r_{ij}<R} s_j(t)\right].$$

Show that updating any single cell, according to the state update rule, cannot increase this function (that is, $\Delta E \le 0$). What else do you need to show in order to guarantee convergence to a stable state?

Problem 2

Prove, by exhibiting a counter-example, that if synchronous updating is used, then the CA may not reach a stable state.

Hint: Construct a very simple CA, obeying the above state update equation, that cycles between two different states.