Lecture 13

Project 2

- See course website under "Projects / Assignments"
- Due Oct. 25
- This project involves programming as well as some lengthy computations
- Start early!

Digression: Time-Reversibility and the Physical Limits of Computation

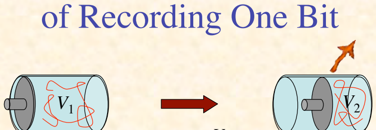
Work done by:

- Rolf Landauer (1961)
- Charles Bennett (1973)
- Richard Feynman (1981–3)

Entropy of Physical Systems

- Recall information content of N equally likely messages: $I_2 = \lg N$ bits.
- Can also use natural logs: $I_e = \ln N$ nats $= I_2 \ln 2$.
- To specify position & momentum of each particle of a physical system: $S = k \ln N = k\, I_2 \ln 2$.
  - k is Boltzmann's constant
- This is the entropy of the system
  - entropies of 10 bits/atom are typical

Thermodynamics of Recording One Bit



$$\Delta S = k \ln \frac{V_2}{V_1}$$

if $V_2 = V_1/2$, then $\Delta S = -k \ln 2$

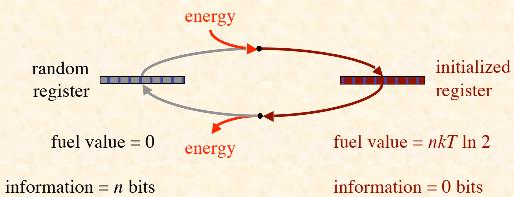
also, $\Delta F = kT \ln 2$

- ΔS derived by gas laws & classical thermodynamics
- Boltzmann constant: $k = 1.381 \times 10^{-23}$ J K^{-1}

Entropy Change in Terms of Phase Space

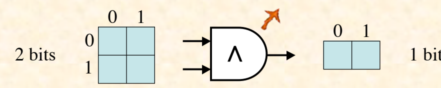
- Let W = number of microstates corresponding to a macrostate
- Entropy $S = k \ln W$
- Then $\Delta S = k \ln W_2 - k \ln W_1 = k \ln (W_2 / W_1)$
- If $W_2 = W_1 / 2$, then $\Delta S = -k \ln 2$

Information and Energy



- initialization equivalent to storing energy
- information and energy are complementary

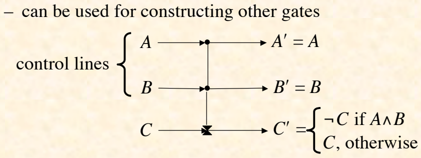
Minimum Energy for Irreversible Computation



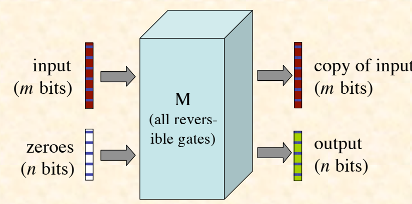
- Loss of one bit of information
  - an irreversible operation (many to one)
- $\Delta S = -k \ln 2$
  - entropy decrease must be compensated by heat dissipation
- Minimum energy required: $\Delta F = kT \ln 2$
  - transistors: ~10^8 kT; RNA polymerase: ~100 kT

Reversible Gates

- Can make dissipation arbitrarily small by using reversible gates
- All outputs must go somewhere
- Cannot *ever* throw information away
- The Fredkin CCN gate ("Controlled Controlled Not") is reversible
  - can be used for constructing other gates



Reversible Computer

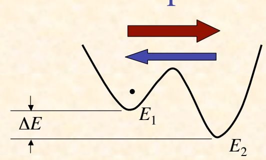


- Reversible because get input back
- Only loss is resetting machine for next job
  - energy is proportional to n, number of output bits

Summary: Energy Required for Reversible Computing

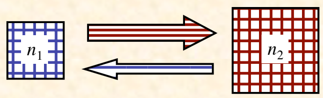
- There is no lower limit on the energy required for basic operations (gates, bit copying, etc.) provided:
  - it is done reversibly
  - it is done sufficiently slowly
- What is the fundamental relation between speed and energy dissipation?

Energy and the Speed of Computation



- Let r be ratio of forward to backward rate
- Statistical mechanics shows: $kT \ln r = \Delta E$
- Greater "driving energy" ⇒ greater rate

Entropy and the Speed of Computation



- Consider number of accessible microstates, n_1 and n_2
- Can show: $r = n_2 / n_1$
- Hence, $kT \ln r = kT (\ln n_2 - \ln n_1)$
  $= (S_2 - S_1)T = T\Delta S$

10/4/07 13

Conclusions

- Entropy increase and energy dissipation can be made arbitrarily small by doing reversible computation
- However, the speed of computation is an exponential function of the driving energy or entropy increase

10/4/07 14

Additional Information

1. Feynman, R. P. *Feynman Lectures on Computation*, ed. by A.J.G Hey & R.W. Allen. Perseus, 1996.
2. Hey, A.J.G. (ed.) *Feynman and Computation: Exploring the Limits of Computation*. Perseus, 1999.

10/4/07 15

Real Ants

(especially the black garden ant, *Lasius niger*)

10/4/07 16

Adaptive Significance

- Selects most profitable from array of food sources
- Selects shortest route to it
  - longer paths abandoned within 1–2 hours
- Adjusts amount of exploration to quality of identified sources
- Collective decision making can be as accurate and effective as some vertebrate individuals

10/4/07 17

Observations on Trail Formation

- Two equal-length paths presented at same time: ants choose one at random
- Sometimes the longer path is initially chosen
- Ants may remain "trapped" on longer path, once established
- Or on path to lower quality source, if it's discovered first
- But there may be advantages to sticking to paths
  - easier to follow
  - easier to protect trail & source
  - safer

10/4/07 18

Process of Trail Formation

1. Trail laying
2. Trail following

Trail Laying

- On discovering food, forager lays chemical trail while returning to nest
  - only ants who have found food deposit pheromone
- Others stimulated to leave nest by:
  - the trail
  - the recruitor exciting nestmates (sometimes)
- In addition to defining trail, pheromone:
  - serves as general orientation signal for ants outside nest
  - serves as arousal signal for ants inside

Additional Complexities

- Some ants begin marking on return from discovering food
- Others on their first return trip to food
- Others not at all, or variable behavior
- Probability of trail laying decreases with number of trips

Frequency of Trail Marking

- Ants modulate frequency of trail marking
- May reflect quality of source
  - hence more exploration if source is poor
- May reflect orientation to nest
  - ants keep track of general direction to nest
  - and of general direction to food source
  - trail laying is less intense if the angle to homeward direction is large

Trail Following

- Ants preferentially follow stronger of two trails
  - show no preference for path they used previously
- Ant may double back, because of:
  - decrease of pheromone concentration
  - unattractive orientation

Probability of Choosing One of Two Branches

- Let C_L and C_R be units of pheromone deposited on left & right branches
- Let P_L and P_R be probabilities of choosing them
- Then:

$$P_L = \frac{(C_L + 6)^2}{(C_L + 6)^2 + (C_R + 6)^2}$$

- Nonlinearity amplifies probability

Additional Adaptations

- If a source is crowded, ants may return to nest or explore for other sources
- New food sources are preferred if they are near to existing sources
- Foraging trails may rotate systematically around a nest

Pheromone Evaporation

- Trails can persist from several hours to several months
- Pheromone has mean lifetime of 30-60 min.
- But remains detectable for many times this
- Long persistence of pheromone prevents switching to shorter trail
- Artificial ant colony systems rely more heavily on evaporation