Lecture 19

Reading

- Flake, ch. 18 (Natural & Analog Computation)

Variables

$\mathbf{x}_k$ = current position of particle k
$\mathbf{v}_k$ = current velocity of particle k
$\mathbf{p}_k$ = best position found by particle k
$Q(\mathbf{x})$ = quality of position $\mathbf{x}$
g = index of best position found so far
i.e., $g = \text{argmax}_k\, Q(\mathbf{p}_k)$
ϕ_1, ϕ_2 = random variables uniformly distributed over [0, 2]
w = inertia < 1

Velocity & Position Updating

$\mathbf{v}_k{}' = w\,\mathbf{v}_k + \phi_1 (\mathbf{p}_k - \mathbf{x}_k) + \phi_2 (\mathbf{p}_g - \mathbf{x}_k)$
$w\,\mathbf{v}_k$ maintains direction (*inertial* part)
$\phi_1 (\mathbf{p}_k - \mathbf{x}_k)$ turns toward private best (*cognition* part)
$\phi_2 (\mathbf{p}_g - \mathbf{x}_k)$ turns towards public best (*social* part)

$\mathbf{x}_k{}' = \mathbf{x}_k + \mathbf{v}_k{}'$

- Allowing $\phi_1, \phi_2 > 1$ permits overshooting and better exploration (*important!*)
- Good balance of *exploration* & *exploitation*
- Limiting $\|\mathbf{v}_k\| < \|\mathbf{v}_{\max}\|$ controls resolution of search

Netlogo Demonstration of Particle Swarm Optimization

Run PSO.nlogo

Yuhui Shi's Demonstration of Particle Swarm Optimization

Run www.engr.iupui.edu/~shi/PSO/AppletGUI.html

Improvements

- Alternative velocity update equation:
  $\mathbf{v}_k' = \chi\,[w\,\mathbf{v}_k + \phi_1\,(\mathbf{p}_k - \mathbf{x}_k) + \phi_2\,(\mathbf{p}_g - \mathbf{x}_k)]$
  χ = constriction coefficient (controls magnitude of $\mathbf{v}_k$)
- Alternative neighbor relations:
  - **star**: fully connected (each responds to best of all others; fast information flow)
  - **circle**: connected to K immediate neighbors (slows information flow)
  - **wheel**: connected to one axis particle (moderate information flow)

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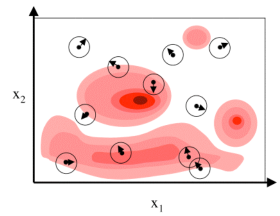
Spatial Extension



- Spatial extension avoids premature convergence
- Preserves diversity in population
- More like flocking/schooling models

10/30/07 Fig. from EVALife site 8

Some Applications of PSO

- integer programming
- minimax problems
  - in optimal control
  - engineering design
  - discrete optimization
  - Chebyshev approximation
  - game theory
- multiobjective optimization
- hydrologic problems
- musical improvisation!

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Millonas' Five Basic Principles of Swarm Intelligence

1. *Proximity principle:*
   pop. should perform simple space & time computations
2. *Quality principle:*
   pop. should respond to quality factors in environment
3. *Principle of diverse response:*
   pop. should not commit to overly narrow channels
4. *Principle of stability:*
   pop. should not change behavior every time env. changes
5. *Principle of adaptability:*
   pop. should change behavior when it's worth comp. price

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Kennedy & Eberhart on PSO

"This algorithm belongs ideologically to that philosophical school

that allows wisdom to emerge rather than trying to impose it,

that emulates nature rather than trying to control it,

and that seeks to make things simpler rather than more complex.

Once again nature has provided us with a technique for processing information that is at once elegant and versatile."

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Additional Bibliography

1. Camazine, S., Deneubourg, J.-L., Franks, N. R., Sneyd, J., Theraulaz, G.,& Bonabeau, E. *Self-Organization in Biological Systems.* Princeton, 2001, chs. 11, 13, 18, 19.
2. Bonabeau, E., Dorigo, M., & Theraulaz, G. *Swarm Intelligence: From Natural to Artificial Systems.* Oxford, 1999, chs. 2, 6.
3. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology.* Basic Books, 2000, ch. 6.
4. Resnick, M. *Turtles, Termites, and Traffic Jams: Explorations in Massively Parallel Microworlds.* MIT Press, 1994, pp. 59-68, 75-81.
5. Kennedy, J., & Eberhart, R. "Particle Swarm Optimization," *Proc. IEEE Int'l. Conf. Neural Networks* (Perth, Australia), 1995. http://www.engr.iupui.edu/~shi/pso.html.

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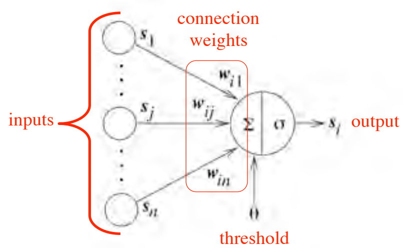
IV. Natural & Analog Computation

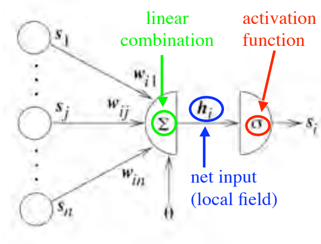
Artificial Neural Networks

(in particular, the Hopfield Network)

Typical Artificial Neuron



Typical Artificial Neuron



Equations

Net input: $$h_i = \left(\sum_{j=1}^{n} w_{ij} s_j \right) - \theta$$

$$\mathbf{h} = \mathbf{W}\mathbf{s} - \theta$$

New neural state: $$s'_i = \sigma(h_i)$$

$$\mathbf{s}' = \sigma(\mathbf{h})$$

Hopfield Network

- Symmetric weights: $w_{ij} = w_{ji}$
- No self-action: $w_{ii} = 0$
- Zero threshold: $\theta = 0$
- Bipolar states: $s_i \in \{-1, +1\}$
- Discontinuous bipolar activation function:

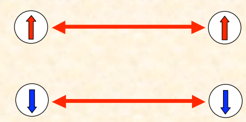
$$\sigma(h) = \operatorname{sgn}(h) = \begin{cases} -1, & h < 0 \\ +1, & h > 0 \end{cases}$$

What to do about $h = 0$?

- There are several options:
  - $\sigma(0) = +1$
  - $\sigma(0) = -1$
  - $\sigma(0) = -1$ or $+1$ with equal probability
  - $h_i = 0 \Rightarrow$ no state change ($s_i' = s_i$)
- Not much difference, but be consistent
- Last option is slightly preferable, since symmetric

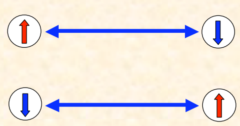
Positive Coupling

- Positive *sense* (sign)
- Large *strength*



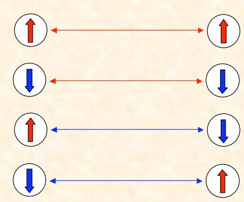
Negative Coupling

- Negative *sense* (sign)
- Large *strength*

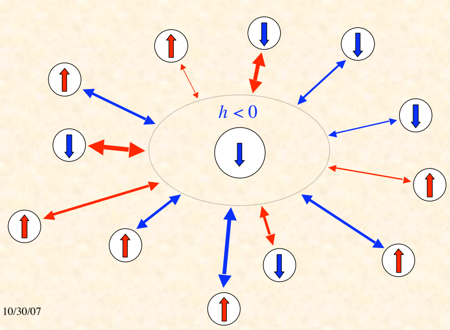


Weak Coupling

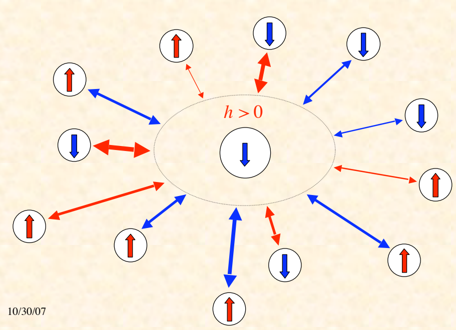
- Either *sense* (sign)
- Little *strength*



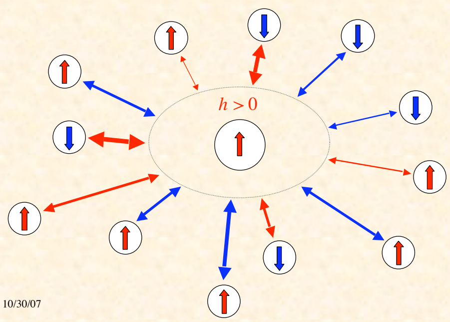
State = –1 & Local Field < 0



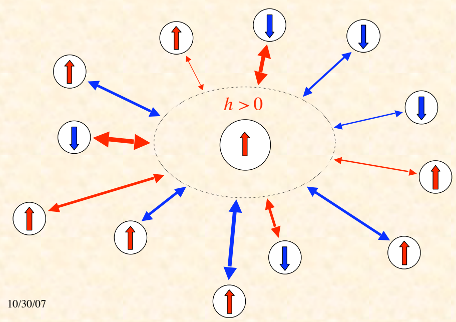
State = –1 & Local Field > 0



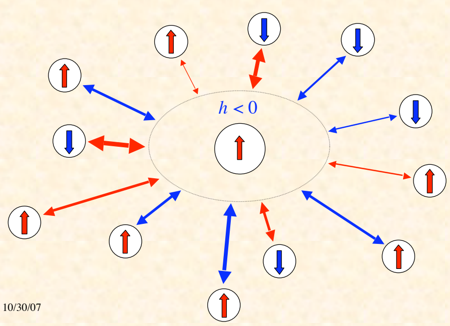
State Reverses



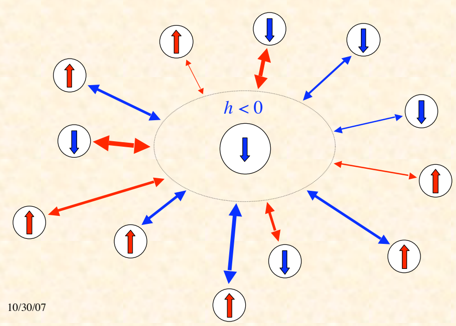
State = +1 & Local Field > 0



State = +1 & Local Field < 0



State Reverses



Hopfield Net as Soft Constraint Satisfaction System

- States of neurons as yes/no decisions
- Weights represent *soft constraints* between decisions
  - *hard* constraints *must* be respected
  - *soft* constraints have *degrees* of importance
- Decisions change to better respect constraints
- Is there an optimal set of decisions that best respects all constraints?

Convergence

- Does such a system converge to a stable state?
- Under what conditions does it converge?
- There is a sense in which each step relaxes the "tension" in the system
- But could a relaxation of one neuron lead to greater tension in other places?