CS311, Spring 2002
Midterm Exam. Monday March 11

Problem 1 True or False. You don’t need to show your work.

(a) \( \neg(p \land q) \iff \neg p \land \neg q \)
(b) \( p \rightarrow q \iff q \rightarrow p \)
(c) \( \neg q \rightarrow \neg p \iff p \rightarrow q \)
(d) \( (p \rightarrow q) \land p \rightarrow q \iff T_0 \)

Problem 2 Choose 2 of the following and prove using the rules of inference. Label each step.

(a) Some turkeys will hold their mouths open in the rain until they drown.
Any animal that drowns itself in the rain is not very smart.
Therefore some turkeys are not very smart.
UNIVERSE = animals.
(b) A dog will grow big if and only if it has big feet as a puppy.
Max is not a big dog.
Therefore Max must not have had big feet as a puppy.
UNIVERSE = dogs.
(c) If a dog is tied up too much it will turn mean.
Tiny is a dog who is not mean.
Therefore Tiny was not tied up too much.
UNIVERSE = dogs.

Problem 3 Prove. State what type of proof you used.

(a) If 2 is divisible by 3 then Elvis is in this room.
(b) For any integer \( n \), \( n(n + 1) \) is even.
(c) \( \sum_{i=0}^{n} \binom{n}{i} = 2^n \). (HINT: This is simple if you remember something that helps. If not, then don’t spend too much time on it.) this right off, then

Problem 4 Answer the following questions. Explain your answers (for partial credit). Multiply the answers out.

(a) How many binary strings of length \( n \) have exactly \( k \) ones?
(b) How many binary strings of length 5 have at least 2 ones?
(c) You have a portable stereo that takes 10 batteries. If 2 of them are dead, how many ways can you replace two batteries, hoping they were the dead ones?
(d) How many "*"s are printed in the code below (in terms of \( n \))?
for (i = 0; i < n; i++)
    for (j = 0; j < i; j++)
        printf("*\n");
printf("  n\n");

(e) How many ways can you make a committee of 6 people from 5 freshman, 3
sophomores, 6 juniors, and 4 seniors if you have to choose one freshman, one sophomore,
2 juniors, and 2 seniors?

Problem 4 Using the binomial theorem, give the coefficient for \(x^4\) in \((2x + 3y)^5\) (you
don’t have to do the whole expansion). Show your work!

Problem 5 Prove by induction \(\sum_{i=0}^{n} i * 2^i = 2^{n+1}(n - 1) - 2\).

Problem 6 Answer the following set questions. Explain your answers.

(a) True or False: If \(A\) is an infinite set and \(B \subseteq A\) then \(B\) is a
finite set.
(b) If \(A = \{a, b, c\}\), what is \(P(A)\) (the power set of \(A\))?
(c) Draw a Venn Diagram showing the following sets:
    \[A = \{x \in \mathbb{Z}^+ | x = 3k \text{ for some integer } k\}\]
    \[B = \{x \in \mathbb{Z}^+ | x \text{ is prime}\}\]
    \[C = \{x \in \mathbb{Z}^+ | x \text{ is even}\}\]
    \[D = \{1, 2, 3, ..., 10\}\]