**CS311, Spring 2002**
**Final Exam. Friday May 3**

**Problem 1** Prove (using rules of inference) or disprove. Label each step.
(a) All dogs have fur.
Animals with fur produce allergies.
Lucky does not cause allergies.
Therefore Lucky is not a dog.
UNIVERSE = animals
(b) If a class is boring then either the material is boring or the professor is boring.
The material in CSXXX is boring.
Therefore the class CSXXX is boring.
UNIVERSE = classes

**Problem 2** Let S = the set of binary strings of length 4. Let R be the relation defined on S such that a R b iff a and b have the same number of ones. For example, $(1100, 0011) \in R$ but $(1100, 1000) \notin R$ because they have different number of ones.
(a) Is R reflexive, symmetric, transitive? Explain why or why not.
(b) Is R an equivalence relation? Explain. If so, list the equivalence classes.

**Problem 3**
(a) Give an example of a function that is 1-1 and onto. Explain why.
Be sure to specify the domain and range, using appropriate notation.
(b) Give an example of a function that is 1-1 but not onto. No explanation needed.
(c) Give an example of a function that is onto but not 1-1. No explanation needed.

**Problem 4** True or false. No explanation needed.
(a) If $A \cup B = C$ then $C \cap A = A$.
(b) $\emptyset$ is a subset of every set.
(c) $|A \cup B| = |A| + |B|$, if $A$ and $B$ are finite sets.
(d) If $A$ is an infinite set then $\bar{A}$ is a finite set.

**Problem 5** Prove or disprove. Say what type of proof used.
(a) For $n \geq 4$, $log(n!) <= nlogn$.
(b) If 3 is odd and then 10 is even.
(c) If $a|b$ and $b|c$ then $a|c$.
Recall $a|b$ means there exists an integer $k$ such that $a * k = b$.
(d) For any real number $x$ $\lceil x \rceil > \lfloor x \rfloor$.

**Problem 6** Big-O problems. Justify your answers.
(a) Show that $f(n) = 2n^2 + 3nlogn + 4n + 1$ is $O(n^2)$.

(b) Show that $f(n) = \binom{n}{3}$ is $O(n^3)$.
(c) Find the best bound for $f(n) = 2log_2 n + 3n - 2$ and show that it is $\Theta$ of that function.

**Problem 7** Answer the following questions. No explanations needed.
(a) T or F: If $f(n)$ is $O(n)$ then $f(n)$ is $O(n^2)$.
(b) T or F: If $f(n)$ is $O(n)$ then $f(n)$ is $\Omega(n^2)$.
(c) Fill in the blank: If $f(n)$ is $O(n)$ then $h(n) = 2 * f(n)$ is $O(-BLANK-)$.
(d) Fill in the blank: If $f(n)$ is $O(n)$ then $h(n) = n * f(n)$ is $O(-BLANK-)$.
Extra Credit: Fill in the blank: If $f(n)$ is $O(n)$ then $h(n) = n + f(n)$ is $O(-BLANK-)$.

**Problem 8** Explain your answers.
(a) How many binary strings of length 5 start with 1 *or* end with 0.
(b) You and 9 friends are going on a trip and plan to take 2 cars. Each car holds at most 6 people. How many ways can you put people in two groups to go into the two cars?
(c) How many binary strings of length 6 have at most three 1's?
(d) How many outfits can you make from 3 pairs of pants, 5 shirts, and 3 jackets if the jacket is optio
(e) How many "words" can you make from 3 consonants and 1 vowel? Assume that there are 5 vowels and 21 consonants. As in English, repeats are allowed.
Extra Credit (2 points) : If you listed the "words" in lexicographic order (abbb, abbc, abcb,...), what is the first string that would be a real word?

**Problem 9** Recurrence relation problems:
(a) Consider the following recurrence relation:

$t(1) = 1$
$t(n) = t(n-1) + 2n$

Guess a formula for $t(n)$.
(b) Consider the following recurrence relation:

$t(1) = 1$
$t(n) = t(\lceil n/2 \rceil) + \lfloor n/2 \rfloor$.

Prove by induction that $t(n) = n$.
(c) Consider the following code:

```
void silly(int n)
{
    int i;
    if (i > 0) {
        for (i = 1; i <= n; i++) printf(".");
        printf(" n");
        silly(n-1);
    }
}
```

Write a recurrence relation $t(n)$ to denote the number of "."'s printed out when "silly" is called with integer $n$.