CS311, Fall 2003
Quiz 1. Tuesday, September 16, 2003

Question 1 (20 points) Answer the following questions. Justify your answers and multiply out any numeric answers.
(a) A puzzle has 8 sliding square pieces that must go in 9 positions (a 3x3 grid) to make a picture.
   How many ways can the pieces be arranged if the ”hole” must be at the top left and 3 of the pieces are identical?
(b) How many binary strings of length \( n \) contain exactly \( k \) ones (in terms of \( k \) and \( n \))?
(c) How many binary strings have length less than or equal to 4?
(d) How many ways can you arrange 5 boys and 4 girls in a line in boy-girl-boy-girl... order?
(e) You have 20 $1 bills which you would like to split among 3 young relatives.

   How many ways can you do this?

Question 2 (15 points) Using logical laws, simplify \((p \lor q) \rightarrow (\neg p \lor \neg q)\) to \(\neg p \lor \neg q\)
As much as you can, label the steps with the name of the law used.

Question 3 (15 points) What is the coefficient of \(x^2y^3\) in \((2x - y)^5\)?

Question 4 (10 points) True or False. Explain your answer in words.
(a) \(((p \rightarrow q) \land p) \rightarrow q \iff T_0\)
(b) \(p \rightarrow q \iff q \rightarrow p\)

Question 5 (40 points) Prove or disprove. If you prove using rules of inference, label the steps as much as you can.
(a) If Jane goes back to school then she will be very busy.
   If Jane is very busy then she will not be able to coach soccer.
   Jane is not able to coach soccer.
   Therefore Jane went back to school.
(b) If my program doesn’t work then either it has a bug or the compiler has an error.
   My program doesn’t work.
   The compiler does not have an error.
   Therefore there is a bug in my program.

Extra Credit (3 points) Show that \(\sum_{i=0}^{n} \binom{n}{i} = 2 \cdot \sum_{i=0}^{n-1} \binom{n-1}{i}\).
Hint: This is sort of a ”trick” problem. It doesn’t involve heavy algebra.