CS 311: Discrete Structures
Spring 2004

Homework 4. Due Thursday, February 12

1. For each of the expressions below, write in English, say whether it is true or false, and explain why. The universe is all integers.
   Assume the following predicates:
   \[ E(x) = \text{"x is even"} \]
   \[ O(x) = \text{"x is odd"} \]
   \[ F(x) = \text{"x is divisible by 4"} \]
   \[ T(x) = \text{"x is divisible by 3"} \]
   (a) \[ \forall x [E(x) \lor O(x)] \]
   (b) \[ \exists x [-F(x) \land E(x)] \]
   (c) \[ \neg \forall x [E(x) \land T(x)] \]
   (d) \[ \forall x [E(x) \land \neg T(x)] \]

2. Let \( U = 1, 2, 3 \) and \( E(x) = \text{"x is even"} \). For each of the following logical expressions: expand it to a logical expression without quantifiers and evaluate it and also write what the expression says in English.
   Example: \( \forall x [E(x)] = E(1) \land E(2) \land E(3) = F \land T \land F = F \)
   In words: All integers in \( U \) are even. This is false.
   (a) \( \neg \forall x [E(x)] \)
   (b) \( \forall x [\neg E(x)] \)
   (c) \( \exists x [E(x)] \)
   (d) \( \neg \exists x [E(x)] \)
   (e) \( \exists x [\neg E(x)] \)

3. For each of the arguments given below, write it using logical notation and say whether it is true or false. If true, give a proof.
   (a) Some little dogs are aggressive.
   All aggressive dogs are scary.
   Therefore some little dogs are scary.
   \[ \text{Universe} = \text{dogs} \]
   (b) All professors are absent-minded.
   Some absentminded people lose their glasses.
   I am a professor.
   Therefore I lose my glasses.
   \[ \text{Universe} = \text{people} \]
   (c) All humans have hearts.
All creatures with hearts have feelings. Therefore all humans have feelings.

Universe = creatures

(d) Some snakes kill rodents.
   Animals that kill rodents are useful.
   Useful animals are good.
   Therefore some snakes are good.

Universe = animals