CS 311: Discrete Structures
Fall 2003

Homework 6. Due Thursday, February 26, 2004

**Question 1** Use set builder notation to describe the following sets:
(a) The set of even positive integers less than 10.
(b) The set of real numbers between 0.5 and 10.5.
(c) The set of integers whose squares are greater than 10.

**Question 2** Let \( A = \{ x \in \mathbb{Z}^+ | x \text{ is odd} \} \) and \( B = \{ x \in \mathbb{Z}^+ | x \text{ a multiple of 4} \} \) and let \( C = \{ x \in \mathbb{Z}^+ | x \leq 10 \} \)
where \( U = \mathbb{Z}^+ \). Determine the following (use proper set notation for your answer):
(a) \( A \)
(b) \( B \cap C \)
(c) \( B \cap A \)
(d) \( (C \cap A) \cap B \)
(e) \( (A \cup B) \cap C \)

**Question 3** Draw a Venn diagram for sets \( A, B, C \) from problem 2.

**Question 4** True or false. Explain.
(Note: \( P(A) \) represents the power set of \( A \))
(a) \( |A \cup B| \leq |A| + |B| \)
(b) \( \emptyset = \{\emptyset\} \)
(c) If \( A \) is infinite the \( A \) is finite.
(d) \( \emptyset \subseteq \{0\} \)
(e) \( \emptyset \in \{\} \)
(f) \( \emptyset \in \{\{\}\} \)

**Question 5** What is the power set of:
(a) \( \{a, b, c, b\} \)
(b) \( \{\emptyset, \emptyset\} \)

**Question 6** Let \( B_i = \{\text{binary strings of length } i \} \)
(a) List the strings in: \( \bigcup_{i=1}^{3} B_i \)
(b) What is $|B_n|$ (as a function of $n$)? (c) Describe in words the set $\cup_{i=1}^n B_i$.
(d) Describe in words the set $\cup_{i=1}^\infty B_i$

**Extra Credit**

Let $B_{i,j}$ = the set of binary strings of length $i$ with exactly $j$ ones ($0 \leq j \leq i$).

(a) List the strings in $B_{3,2}$
(b) What is $|B_{1,1}|$, $|B_{2,2}|$, $B_{n,n}$? Explain.
(c) Give an expression for $|B_{i,j}|$.
(d) Using $\cup_{i=\infty}$ notation and $B_{i,j}$, give an expression for the set of all binary strings of length 5.
(e) What is $\cup_{i=1}^\infty B_{i,0}$?