CS311, Spring 2004
Quiz 1. Tuesday, February 10, 2004

Question 1 20 points Answer the following questions. Justify your answers and
multiply out any numeric answers. Show your work.
(a) How many binary strings of length 6 have exactly 4 ones?
(b) How many ways can you put 10 identical computers into 4 offices if each office
must have at least one computer?
(c) A combination lock has 3 rotating disks with the digits 0,..., 9. How many possible
combinations are there if the first digit is either a 1 or a 2 and the 2nd digit is not a 6?
(d) A child has 7 different stuffed teddy bears. How many ways can she choose 4 to give away?
(e) How many subsets of the set \{1, 2, \ldots, n\} contain exactly \(k\) items?

Question 2 20 points For each of the following, say whether it is a tautology
or not and justify your answer. If it is a tautology, use logical laws to show.
(a) \(\neg(p \lor (p \rightarrow q)) \rightarrow r\)
(b) \(p \rightarrow (q \rightarrow r) \Leftrightarrow (p \rightarrow q) \rightarrow r\)

Question 3 15 points What is the coefficient of \(x^3y\) in \((x - 2y)^4\)?

Question 4 45 points Prove or disprove. If you prove using rules of inference,
label the steps as much as you can.
(a) If I exercise and diet then I will be able to lose weight.
I am able to lose weight.
Therefore I exercise and diet.
(b) If my dog is sick then I will take him to the vet.
If I take my dog to the vet, then he has to ride in the car.
If my dog has to ride in the car, then he is unhappy.
My dog is sick.
Therefore he will be unhappy.
(c) If my dog gets to swim in the lake or chase a ball, then he is happy.
If my dog gets to swim in the lake and chase a ball, then he is ecstatic.
My dog is not happy.
Therefore my dog is not ecstatic.

Extra Credit: A game of chance has a tic-tac-toe board where each location
contains a digit between 0 and 9. You win if you get 3 7's in a row. How many
possible combinations are there? How many of them are winning combinations? What
is the probability of winning? (The probability is the number of ways of winning divided
by the number of possibilities).