CS311, Spring 2004  
Quiz 2. Thursday, April 15, 2004

Question 1 Consider the recurrence relation:
\[ t(n) = 2t(n/2) + 2n \]  
\[ t(1) = 1. \]  
(a) Compute \( t(2), t(4), \) and \( t(8). \)  
(b) Prove by induction that \( t(n) = 2n \log n + 2n \) for \( n \) a power of 2.

Question 2 Consider the recurrence relation:
\[ t(n) = 2t(n - 1) + 3 \]  
\[ t(0) = 1. \]  
(a) Compute \( t(1), t(2), \) and \( t(3). \) Show your work.  
(b) Guess a closed-form solution for \( t(n). \)  
Extra Credit: Use unrolling to derive a closed-form solution for \( t(n). \)

Question 3 Choose and answer 2. For two of the recursively defined sets below, show one string in the set and one string not in the set. Also describe the set in words, describing clearly what it does contain (and what it does not contain).
(i) \( S \) is a set of strings of 0’s and 1’s defined by the following rules:
   \[ 01 \in S. \]
   If \( x \in S \) then \( 01x \) is in \( S. \)
   No strings are in \( S \) except strings formed by these rules.
(ii) \( S \) is a set of integers.
   \[ 0 \in S. \]
   If \( x \in S \) then \( x + 5 \in S. \)
   No integers are in \( S \) except those formed by these rules.
(iii) \( S \) is a set of binary strings.
   \[ 00 \in S \]
   \[ 11 \in S. \]
   If \( x \in S \) then \( 0x0 \in S. \)
   If \( x \in S \) then \( 1x1 \in S. \)
   No strings are in \( S \) except strings formed by these rules.

Question 4 Prove: If \( a, b, \) and \( d \) are positive integers, then if \( d|a \) and \( d|b \) then \( d|a + b. \)

For the remaining questions, choose and answer any TWO.

Question 5 True or False.
(a) If \( A \cup C = B \cup C \) then \( A = B. \)
(b) If set \( A \) is finite and we know \( |P(A)| \) then we can compute \( |A|. \)
(\(P(A)\) is the power set of \(A\).)

**Question 6** Draw a Venn diagram showing sets \(A, B,\) and \(C\) where the universe \(U = \mathbb{Z}^+ \cup \{0\},\) and \(A = \{x \in U \mid x \text{ is prime}\}\) and \(B = \{x \in U \mid x \text{ is odd}\}\) and \(C = \{x \in U \mid 0 \leq x \leq 20\}\).

**Question 7** Let \(B_i = \) the set of binary strings of length \(i\). For example, \(B_1 = \{0, 1\}\) and \(B_2 = \{00, 01, 10, 11\}\).
(a) Give a formula for \(|B_n|\) in terms of \(n\).
(b) Describe the set \(\bigcup_{i=1}^n B_i\).
(c) Describe the set \(\bar{B}_i\).

**Question 8** Euclid’s GCD algorithm:
(a) Calculate the \(\gcd(385, 75)\). Show your work.
(b) How many divisions (calculating the mod) were required in part (a)?
**Extra Credit:** The alternative method for computing the gcd would involve computing the prime factorization of the two numbers. For 385 and 75, does Euclid’s GCD algorithm require fewer divisions than computing the prime factorization of both numbers?