CS494, Fall 2002
Homework 1. Due Monday September 9, 2002

**Problem 1** Consider the following recurrence relation:
\[ t(n) = 2 \times t(n/2) + n \]
\[ t(1) = 2. \]
Prove by induction that \( t(n) = O(n \log_2 n) \).

**Problem 2** Consider the following algorithm for computing the convex hull of a set of \( n \) points.

Step 1. Divide the points into two sets of size \( n/2 \) (in no particular order)
Call these sets \( S_1 \) and \( S_2 \).
Step 2. Recursively compute the convex hull \( H_1 \) of \( S_1 \) and \( H_2 \) of \( S_2 \).
Step 3. Compute the convex hull \( H \) of the whole set from \( H_1 \) and \( H_2 \).
(a) Assuming that Step 3 takes \( 2n \) time, write a recurrence relation for this algorithm.
(b) Guess a closed form for the recurrence relation.
Extra Credit: Prove that the closed form is correct.

**Problem 3** Consider the following algorithm for finding the convex hull of a set of \( n \) points.

Step 1. Sort the points by increasing \( x \)-coordinate.
Step 2. Recursively compute the convex hull of the left half of the points.
   Call this result \( H_1 \).
Step 3. Recursively compute the convex hull of the right half of the points.
   Call this result \( H_2 \).
Step 4. From \( H_1 \) and \( H_2 \) compute the convex hull \( H \) of the whole point set.

(a) Consider this possible approach to doing step 4:
   Find the highest and lowest points in \( H_1 \) and \( H_2 \).
   Add an edge connecting the highest points in \( H_1 \) and \( H_2 \).
   Add an edge connecting the lowest points in \( H_1 \) and \( H_2 \).
   Delete all the points that are now inside.
Does this always work? Why or why not? If not, give an example.

(b) Call the chain of edges on \( H_1 \) from top to bottom along the right side
   the ”inner chain of \( H_1 \)”.
   Similarly call the chain of edges from the top to the bottom of \( H_2 \) along the left side the inner chain of \( H_2 \).
Can any of the points along the inner chains (except the highest and lowest) be in
the convex hull of the whole point set? Why or why not?

(c) Explain how to correctly find the convex hull of \( H \) from \( H_1 \) and \( H_2 \).

**Problem 4** Write code in C (on paper) to perform the slow algorithm for
finding the convex hull given in the book on page 6. It does not have
to run, but it should be close.