

# Chapter 10 :: Functional Languages

## *Programming Language Pragmatics*

---

Michael L. Scott

# Historical Origins

- The imperative and functional models grew out of work undertaken Alan Turing, Alonzo Church, Stephen Kleene, Emil Post, etc. ~1930s
  - different formalizations of the notion of an algorithm, or *effective procedure*, based on automata, symbolic manipulation, recursive function definitions, and combinatorics
- These results led Church to conjecture that *any* intuitively appealing model of computing would be equally powerful as well
  - this conjecture is known as *Church's thesis*

# Historical Origins

- Mathematicians established a distinction between
  - *constructive* proof (one that shows how to obtain a mathematical object with some desired property)
  - *nonconstructive* proof (one that merely shows that such an object must exist, e.g., by contradiction)

# Historical Origins

- Turing's model of computing was the *Turing machine* a sort of pushdown automaton using an unbounded storage “tape”
  - the Turing machine computes in an imperative way, by changing the values in cells of its tape – like variables just as a high level imperative program computes by changing the values of variables

# Historical Origins

- Church's model of computing is called the *lambda calculus*
  - based on the notion of parameterized expressions (with each parameter introduced by an occurrence of the letter  $\lambda$ —hence the notation's name.
  - Lambda calculus was the inspiration for functional programming
  - one uses it to compute by substituting parameters into expressions, just as one computes in a high level functional program by passing arguments to functions

# Functional Programming Concepts

- Functional languages such as Lisp, Scheme, FP, ML, Miranda, and Haskell are an attempt to realize Church's lambda calculus in practical form as a programming language

# Functional Programming Concepts

- Key Idea: Define the outputs of a program as a mathematical function of the inputs
  - No mutable state
  - No side-effects
  - Emphasizes recursion rather than iteration

# Origins of Functional Languages

- AI modules for game playing in the 1950s
- Branch-and-bound algorithms ideally suited for recursion



# Functional Programming Concepts

- Significant features, many of which are missing in some imperative languages
  - 1st class and high-order functions
  - implicit, parametric polymorphism
  - powerful list facilities
  - structured function returns
  - fully general aggregates
  - garbage collection

# Functional Programming Concepts

- So how do you get anything done in a functional language?
  - Recursion (especially tail recursion) takes the place of iteration
  - In general, you can get the effect of a series of assignments

```
x := 0      ...  
x := expr1  ...  
x := expr2  ...
```

from  $f3 (f2 (f1 (0) ) )$ , where each  $f$  expects the value of  $x$  as an argument,  $f1$  returns  $expr1$ , and  $f2$  returns  $expr2$

# Functional Programming Concepts

- Recursion even does a nifty job of replacing looping

```
x := 0; i := 1; j := 100;
while i < j do
    x := x + i*j; i := i + 1;
    j := j - 1
end while
return x
```

becomes  $f(0, 1, 100)$ , where

```
f(x, i, j) == if i < j then
f(x+i*j, i+1, j-1) else x
```

# Natural Recursive Problems

- Recurrences

- E.g., factorial:

$$0! = 1$$

$$1! = 1$$

$$n! = n * (n-1)!$$

- E.g., greatest common divisor

```
int gcd(int a, int b) {  
    if (a == b) return a;  
    else if (a > b) return gcd(a - b, b);  
    else return gcd(a, b - a);  
}
```

- Tree traversals

- Graph traversals

# Speeding Up Recursion

- Tail recursion: Recursion in which additional computation never follows a recursive call
- Compiler optimizes a tail recursive function by re-using the stack frame for the function
- Example

```
int gcd(int a, int b) {  
    start: if (a == b) return a;  
           else if (a > b) { a = a - b; goto start; }  
           else { b = b - a; goto start; }  
}
```

# Transforming Recursion to Use Tail Recursion

- *continuations*: arguments that contain intermediate results which get passed to successive recursive calls
- Example

```
factorial(n, product) {  
    if (n == 0 or n == 1) return product  
    else  
        return factorial(n-1, product * n) }
```

# Continuations are like imperative programming

- Example: The fibonacci recurrence relation

$$\text{fib}_0 = 0$$

$$\text{fib}_1 = 1$$

$$\text{fib}_n = \text{fib}_{n-1} + \text{fib}_{n-2}$$

- a natural functional implementation:

```
fib(n) {  
    if (n == 0) return 0  
    else if (n == 1) return 1  
    else return fib(n-1) + fib(n-2)  
}
```

# Continuations (cont)

- An efficient C implementation

```
int fib(int n) {  
    int f1 = 0; f2 = 1;  
    int i;  
    for (i = 2; i <= n; i++) {  
        int temp = f1 + f2;  
        f1 = f2;  
        f2 = temp;  
    }  
    return f2;  
}
```



# Continuations (cont)

- A functional implementation of fib using continuations

```
fib(n) {  
  fib-helper(f1, f2, i) {  
    if (i == n) return f2;  
    else return fib-helper(f2, f1 + f2, i+1);  
  }  
  return fib-helper(0, 1, 0);  
}
```

# Functional Programming Concepts

- Lisp also has (these are not necessary present in other functional languages)
  - homo-iconography
  - self-definition
  - read-evaluate-print
- Variants of LISP
  - Pure (original) Lisp
  - Interlisp, MacLisp, Emacs Lisp
  - Common Lisp
  - Scheme

# Functional Programming Concepts

- Pure Lisp is purely functional; all other Lisps have imperative features
- All early Lisps dynamically scoped
  - Not clear whether this was deliberate or if it happened by accident
- Scheme and Common Lisp statically scoped
  - Common Lisp provides dynamic scope as an option for explicitly-declared *special* functions
  - Common Lisp now THE standard Lisp
    - Very big; complicated (The Ada of functional programming)

# Functional Programming Concepts

- Scheme is a particularly elegant Lisp
- Other functional languages
  - ML
  - Miranda
  - Haskell
  - FP
- Haskell is the leading language for research in functional programming

# A Review/Overview of Scheme

- Scheme is a particularly elegant Lisp
  - Interpreter runs a read-eval-print loop
  - Things typed into the interpreter are evaluated (recursively) once
  - Anything in parentheses is a function call (unless quoted)
  - Parentheses are NOT just grouping, as they are in Algol-family languages
    - Adding a level of parentheses changes meaning

# A Review/Overview of Scheme

## Example program - Simulation of DFA

- We'll invoke the program by calling a function called 'simulate', passing it a DFA description and an input string
  - The automaton description is a list of three items:
    - start state
    - the transition function
    - the set of final states
  - The transition function is a list of pairs
    - the first element of each pair is a pair, whose first element is a state and whose second element is an input symbol
    - if the current state and next input symbol match the first element of a pair, then the finite automaton enters the state given by the second element of the pair

# A Review/Overview of Scheme

## Example program - Simulation of DFA

```
(define simulate
  (lambda (dfa input)
    (cons (current-state dfa)          ; start state
          (if (null? input)
              (if (infinal? dfa) '(accept) '(reject))
              (simulate (move dfa (car input)) (cdr input))))))

;; access functions for machine description:
(define current-state car)
(define transition-function cadr)
(define final-states caddr)
(define infinal?
  (lambda (dfa)
    (memq (current-state dfa) (final-states dfa))))

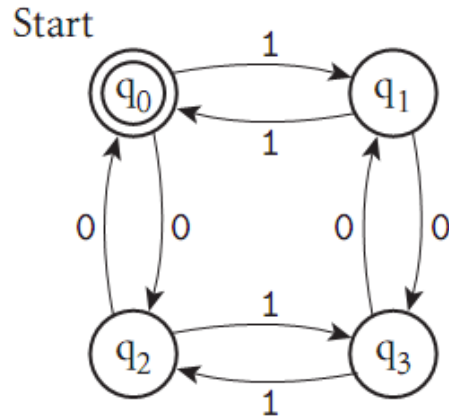
(define move
  (lambda (dfa symbol)
    (let ((cs (current-state dfa)) (trans (transition-function dfa)))
      (list
        (if (eq? cs 'error)
            'error
            (let ((pair (assoc (list cs symbol) trans)))
              (if pair (cadr pair) 'error))) ; new start state
        trans ; same transition function
        (final-states dfa)))) ; same final states
```

**Figure 10.1** Scheme program to simulate the actions of a DFA. Given a machine description and an input symbol *i*, function `move` searches for a transition labeled *i* from the start state to some new state *s*. It then returns a new machine with the same transition function and final states, but with *s* as its “start” state. The main function, `simulate`, tests to see if it is in a final state. If not, it passes the current machine description and the first symbol of input to `move`, and then calls itself recursively on the new machine and the remainder of the input. The functions `cadr` and `caddr` are defined as `(lambda (x) (car (cdr x)))` and `(lambda (x) (car (cdr (cdr x))))`, respectively. Scheme provides a large collection of such abbreviations.



# A Review/Overview of Scheme

## Example program - Simulation of DFA



```
(define zero-one-even-dfa
  '(q0 ; start state
    (((q0 0) q2) ((q0 1) q1) ((q1 0) q3) ((q1 1) q0) ; transition fn
      ((q2 0) q0) ((q2 1) q3) ((q3 0) q1) ((q3 1) q2))
    (q0))) ; final states
```

**Figure 10.2** DFA to accept all strings of zeros and ones containing an even number of each. At the bottom of the figure is a representation of the machine as a Scheme data structure, using the conventions of Figure 10.1.



# Evaluation Order Revisited

- Applicative order
  - what you're used to in imperative languages
  - usually faster
- Normal order
  - like call-by-name: don't evaluate arg until you need it
  - sometimes faster
  - terminates if anything will (Church-Rosser theorem)

# Evaluation Order Revisited

- In Scheme
  - functions use applicative order defined with lambda
  - special forms (aka macros) use normal order defined with syntax-rules
- A *strict* language requires all arguments to be well-defined, so applicative order can be used
- A *non-strict* language does not require all arguments to be well-defined; it requires normal-order evaluation

# Evaluation Order Revisited

- Lazy evaluation gives the best of both worlds
- But not good in the presence of side effects.
  - delay and force in Scheme
  - delay creates a "promise"

# High-Order Functions

- Higher-order functions
  - Take a function as argument, or return a function as a result
  - Great for building things
  - Currying (after Haskell Curry, the same guy Haskell is named after)
    - For details see Lambda calculus on CD
    - ML, Miranda, and Haskell have especially nice syntax for curried functions

# Functional Programming in Perspective

- Advantages of functional languages
  - lack of side effects makes programs easier to understand
  - lack of explicit evaluation order (in some languages) offers possibility of parallel evaluation (e.g. MultiLisp)
  - lack of side effects and explicit evaluation order simplifies some things for a compiler (provided you don't blow it in other ways)
  - programs are often surprisingly short
  - language can be extremely small and yet powerful

# Functional Programming in Perspective

- Problems
  - difficult (but not impossible!) to implement efficiently on von Neumann machines
    - lots of copying of data through parameters
    - (apparent) need to create a whole new array in order to change one element
    - heavy use of pointers (space/time and locality problem)
    - frequent procedure calls
    - heavy space use for recursion
    - requires garbage collection
    - requires a different mode of thinking by the programmer
    - difficult to integrate I/O into purely functional model