

CS580 HW1

Due: 2023-08-30, 4:10PM

1. Prove by induction on n that $\sum_{i=1}^n (2i - 1) = n^2$
2. Let R be a relation on the real numbers such that aRb iff (if and only if) $|a - b| < \sigma$, where σ is a very small number. (Recall that a relation is a set of pairs, and aRb is shorthand for saying that the pair (a, b) is in the set R .) Prove or disprove that R is an equivalence relation.
3. Let R be the relation $(1, 2), (2, 3), (3, 4), (5, 4)$ on \mathbb{N} .
 - (a) What is the transitive closure of R ?
 - (b) What is the symmetric closure of R ?
 - (c) What is the cardinality of the transitive and symmetric closure of R ?
4.
 - (a) Give an example of a relation that is symmetric and transitive but not reflexive.
 - (b) Give an example of a relation that is reflexive and transitive but not symmetric
 - (c) Give an example of a relation that is reflexive and symmetric but not transitive
5. Prove or disprove: The set of all ordered pairs of natural numbers is countably infinite.
6. Prove or disprove: The union of a countably infinite collection of countably infinite sets is countably infinite.
7. Prove or disprove: The Cartesian product of a countably infinite collection of countably infinite sets is countably infinite.