CS580 Homework 12

November 22 4:10 PM, 2023

1. A dominating set in a graph G is a set S of vertices such that every vertex in G is either in S or has an edge to a vertex in S. The decision version of the dominating set problem is defined as follows:

   - Input: a graph G and a positive integer k.
   - Output: yes or no, depending on whether G has a dominating set of size k.

   Answer the following:

   a. Define the search version of the dominating set problem.

   b. Define an optimization version of the dominating set problem.

   c. Assume we have algorithm A to solve the decision version of the problem. How would we use A to solve the search version?

   d. How would we use A to solve your optimization version of the problem? .

2 The decision version of the firehouse problem is defined as follows:

   - Input: a graph G, with integer distances on its edges, and two integers f and d.
   - Output: yes or no, depending on whether there exists a way to select f vertices of G on which to locate "firehouses" such that no vertex is at a greater distance than d from the nearest firehouse.

   Show that the firehouse problem is $\mathcal{NP}$-complete by using the fact that the dominating set problem is $\mathcal{NP}$-complete