CS 580 Homework 4

September 20 4:10 PM, 2023

1. Use the Myhill-Nerode theorem to determine which, if any, of these languages are regular. If a language is regular, list its canonical equivalence classes. Otherwise, argue convincingly that its set of canonical equivalence classes is infinite. For all problems let $\sum = \{a, b\}$.

   a. $\{a^{i^2} | i \geqslant 0\}$.

   b. $\{b^i a^j | i \text{ is divisible by three}, j \text{ is odd}\}$.

   c. $\{w | w \text{ has an equal number of } ab \text{ and } ba \text{ substrings}\}$.

   d. $\{b^i a^j b^{i+j} | i, j \geq 1\}$.

   e. $\{ww^R | w \in (a+b)^*, w^R \text{ is the reverse of } w\}$

   f. $\{a^n b^m a^p | n + 2p = 2m\}$

2. Prove each statement using closure properties of regular sets, or give a counterexample.

   a. Regular sets are closed under infinite union.

   b. If L has a non-regular proper subset, then L is non-regular.

   c. Every infinite regular set has infinitely many infinite regular proper subsets.