Tabu Search

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Questions.

1. What’s the general difference between heuristics and metaheuristics?
2. Who is the creator of Tabu search algorithm?
3. VRP is a generalization of what other famous problem?
Presenters
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Hesam Shams
Ph.D. aspirants at the ISE
Andrii Berdnikov
Drove to UT from Chicago in 2015, where I was working as Analytical Consultant (SAS based forecasting solutions).
Andrii Berdnikov

- MS in Applied Mathematics (Interior Point Methods) from Georgia Southern.
- MS in Pure Mathematics (Asymptotic Problems in Analytical Number Theory) from the Odessa Elie Metchnikoff University (among alumni George Gamov, Ivan Sechenov and the first rabbi appointed as a US Navy chaplain).
- Research interests: Conic Optimization, Global Optimization.
- Hobbies: swimming, water polo, history.
Hesam Shams

- Studying Ph.D. in Industrial Engineering
  - Advisor: Sr. Shylo

- Born and raised in Tehran, Iran
  - Applied Math B.Sc.
  - Industrial Engineering M.Sc.

- Research interests:
  - Applied Optimization, Scheduling, Mathematical Programming

- Interests and Hobbies:
  - Photography, Cooking, Movies, Music
Contents

- VRP
  - History
  - Importance
  - Methods

- Tabu search
  - Local Search
  - Heuristics vs Metaheuristics
  - History
  - Memory

- Guided Tabu Algorithm

- Comparison
VRP: History

G. B. Dantzig and J. H. Ramser


Their work was also presented at the Fifth World Petroleum Congress held in New York in 1959.
VRP: History

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George Dantzig was working for Rand Corporation
VRP: History

John Hubert Ramser

Research and Development Department, the Atlantic Refining Company in Philadelphia (later ARCO).
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Published at least one more mathematical paper: "Theory of Thermal Diffusion under Linear Fluid Shear" in Journal of Industrial and Engineering Chemistry (1956).
VRP: History

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Published at least one more mathematical paper: "Theory of Thermal Diffusion under Linear Fluid Shear" in Journal of Industrial and Engineering Chemistry (1956).

One of the founders of SIAM. Was elected among SIAM’s first Council members, October 1952.
VRP: Formulation

Classical VRP is defined on an undirected graph of $n+1$ vertices where vertex 0 is considered to be a depot, and the rest are destination points.
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Classical VRP is defined on an undirected graph of \( n + 1 \) vertices where vertex 0 is considered to be a depot, and the rest are destination points.

With each vertex it is associated and positive demand \( q_i \).

And with each edge \((i, j)\) a positive cost \( c_i \).

\[
\min z = \sum \sum c_{ij} x_{ij}, \text{ s.t.}
\]

\[
\forall i \in V \{0\}: \sum_{j \in V \{i\}} x_{ij} = 1, \sum_{j \in V \{i\}} x_{ji} = 1
\]
VRP: Complexity

The uncapacitated VRP is equal to a multiple-salesman variant of the TSP.
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The uncapacitated VRP is equal to a multiple-salesman variant of the TSP.

As most of the routing problems are extension of VRP they also belong to the class of strongly non-deterministic polynomial time hard (NP-hard) combinatorial optimization problems.
VRP: Variations of Classical problem

- Capacitated Vehicle Routing Problem (CVRP)
- Vehicle Routing Problem with Time Windows (VRPTW)
- Multiple Depot VRP
- Split Delivery VRP
- VRP with Pickup and Delivering
- VRP with Backhauls
- and many many others.
VRP: Importance

Number of publications with regard to VRP. (From Ferucci[1]).
Local Search
Local Search

- Move configurations by local moves
- Works with complete assignments to the decision variables and modifying
- It is not partial assignment and expanding
Local Search

▪ How to work with local search?
  ▪ Satisfaction problems
    ▪ Start with a feasible solution
    ▪ Move in feasible region
  ▪ Pure optimization problems
    ▪ Start with sub-optimal solution
    ▪ Move in sub-optimal solutions
  ▪ Constrained optimization problems
Local Move

- Local move
  - many choices
  - assign a value to a decision variable

- Select a move
  - many choices
  - max/min conflict
Neighborhood

Optimizing a function

Local moves defines neighborhood

Local Search is a graph exploration
Swaps

Neighborhood
  ◦ Swap two configurations

Search strategy
  ◦ Find a configuration that appears in violations
  ◦ Swap the configuration with another configuration to minimize the number of violations

Why swap, not assignment
  ◦ Automatically maintain the constraints
  ◦ Hard constraints
  ◦ Soft constraints
Traveling Salesman Problem (2-OPT)
Local Search

States

- Either solutions or configurations

Moving from state $s$ to one of its neighbors

- $N(s)$: neighbors of $s$

Legal neighbors

- Some of neighbors are legal, others are not
- $L(N(s), s)$: set of legal functions

Select one legal neighbor

- $S(L(N(s), s), s)$: selection function

Objective function

- Minimizing $f(s)$
Graphical Illustration

\[ S(L(N(s))) \]

\[ N(s) \]

\[ L(N(s)) \]
Local Minima

A configuration $c$ is a local minima with neighbors $N$

$$\forall n \in N(c): f(n) \geq f(c)$$

No guarantees for global optimal

- Escaping from local minima is a critical issue in local search
Local Search
High Quality of Local Minima
Heuristics vs Meta-Heuristics

Heuristics
- Choose next neighbors
- Use local information (the configuration and its neighbors)
- Drive search to a local minimum

Meta-heuristics
- Escaping from local minimum
- Drive search to the global minimum
- Include memory or learning
Tabu Search
Tabu Search

Tabu node ●: node that I already visited
Tabu Search - History
by Glover in 1986 and formalized in 1989

The word *tabu* comes from Polynesian culture
- things that cannot be touched

Local search and Tabu list

Maintain the sequence of visited nodes
VRP: Methods

Classification of different solution methods for node-based routing problems (in accordance with Ferucci[1])
VRP: Methods

G. Clarke (affiliated with Wholesale Cooperative Society of Manchester) and J. W. Wright (University of Manchester) introduced savings algorithm in a 1964 paper. It utilizes a greedy approach.
VRP: Heuristic methods

Savings Algorithm.

*Savings of merging two routes:*

\[ S_{12} = c_{1D} + c_{D2} - c_{12} \]
VRP: Heuristic methods

Savings Algorithm (Parallel).

1. Compute the savings for pair of demand nodes.
2. Sort the savings in non-ascending (descending) order.
3. Starting at the top of the list, merge the two routes associated with the largest savings value.
4. Repeat step (3) until no additional savings can be achieved.
VRP: Savings algorithm
VRP: Heuristic methods


1. Set the Depot as a center of coordinates.
2. Construct a route counterclockwise until any of the constraints is violated.
3. Repeat step (2) until all nodes are covered.
4. (Second Phase). Optimize every route solving a TSP.
VRP Heuristic: route-first-cluster-second

1. Construct a giant TSP route disregarding the constraints.

2. Split the cycle into feasible vehicle routes.
VRP Heuristic: cluster-first-route-second

1. Split the graph into clusters each of which would satisfy the feasibility constraints.

2. Optimize route on each cluster by solving a TSP.
VRP: Metaheuristic methods

Higher level algorithms, more sophisticated.

Designed to escape locally optimal solutions

Many of them inspired by physical or biological phenomena.
VRP: Simulated Annealing

1. Given initial solution $S$, $S^* = S$, $T = T_{max}$.

2. $S' = RandomTransformation(S^*)$

3. If $\Delta f < 0$ or $p(\Delta f, T') = e^{\frac{\Delta f}{T'}} < randomP$ then $S = S'$

4. If $S'$ is better than $S^*$ then $S^* = S'$.

5. If the exit criterion is not met then $T = \alpha T$ and GOTO step 2.

6. Return $S^*$. 
VRP: Simulated Annealing

Types of transformations to apply randomly. (According to Kokubugata, Kawashima [2008])
Memory in Tabu Search

Expensive to maintain all visited nodes

Short-term memory
- Only keep a small list of visited nodes (tabu list)
- Increase or decrease it dynamically

Solution abstractions
- Store transitions; not states
- Weakness: cannot prevent cycling
- Strength: swap can produce multiple solutions in the tabu list
Guided Tabu Search

How to make it better?

Fixing Tabu list size in traditional Tabu

Even dynamic Tabu list methods store all components

Guided Tabu?

- Store solutions in binary
- Each binary has a dynamic Tabu tenure list
- Updates dynamically the tenure size (depends on previous solutions)
- Tabu tenure values is approximated by Boltzmann distribution
Applications of Tabu

Graph theory related problems (TSP, Assignment, Logistics, …)

Scheduling problems

Clustering

Discrete optimization problems
Tabu in VRP

- Gendreau et al. apply Tabu on VRP
- Taillard apply a parallel iterative Tabu on VRP
- Many researcher studied Tabu on different versions of VRP
- Many researcher studied a hybrid of Tabu with other heuristics and meta-heuristics on VRP
Benchmark

Taillard proposes 12 instances in his benchmark
- from 75 to 385 customers

Best found solutions are still updating
Comparison

The Probability of classes compared on all problems for 99.9% confidence interval

- GTA04
- Tabu
Open Issues

How to improve storing history by components effectively?
How to abstract the Tabu tenure list?
How to improve the convergence to the global optimal?
Thank You
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