

Sifting Property of Impulse Function $\delta(t)$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\int_{-\infty}^{\infty} f(t) \delta(t) dt = f(\phi)$$

$$\boxed{\int_{-\infty}^{\infty} f(t) \delta(t-t_0) dt = f(t_0)}$$

"sifting Property" of $\delta(t)$
behaves somewhat like a sampler. (more later)

Transfer Functions

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}$$

z_i are called "zeros" \rightarrow where $H(s) = 0$
 p_i are called "poles" \rightarrow where $H(s) \rightarrow \infty$

When our signals & system are all real
If any pole is complex, its complex conjugate is

also a pole (same for zeros)

Transfer function $H(s)$ is a system property & does not depend
on the input or output function.

$V_o(s) = V_i(s) \cdot H(s) \rightarrow$ output will have zeros & poles from both input
& the transfer function

Partial Fraction Expansion / Decomposition

$$\frac{\sum_{i=0}^M a_i s^i}{\sum_{i=0}^N b_i s^i} = \frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)} = \frac{k_1}{(s - p_1)} + \frac{k_2}{(s - p_2)} + \cdots + \frac{k_N}{(s - p_N)}$$

PFE
factoring

First, assume all p_i are unique $\notin M \cup N$ (will do alternate case next)

PFE: to find k_i , multiply both sides by $(s - p_i)$ & evaluate at $s = p_i$

or for k_i :

$$\frac{(s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)} \Big|_{s=p_i} = \left[\frac{k_1}{(s - p_1)} + \frac{k_2}{(s - p_2)} + \cdots + \frac{k_N}{(s - p_N)} \right] \Big|_{s=p_i} = k_i$$

or sometimes called "cover-up" method

$$F(s) = \frac{4(s+2)}{s^2 + 4s + 3} = \frac{4(s+2)}{(s+3)(s+1)} = \frac{k_1}{s+3} + \frac{k_2}{s+1} = \frac{2}{s+3} + \frac{2}{s+1}$$

$$k_1 = \frac{4(s+2)}{(s+1)} \Big|_{s=-3} = 2$$

$$f(t) = L^{-1}\{F(s)\} =$$

$$k_2 = \frac{4(s+2)}{(s+3)} \Big|_{s=-1} = 2$$

$$f(t) = 2e^{-3t} u(t) + 2e^{-t} u(t)$$



PFE: Repeated Roots

$$F(s) = \frac{N(s)}{(s-p_1)(s-p_2)^2} = \frac{k_1}{s-p_1} + \frac{k_2}{(s-p_2)} + \frac{k_3}{(s-p_2)^2} = \frac{k_1}{s-p_1} + \frac{k_2 s + (k_3 - p_2 k_2)}{(s-p_2)^2}$$

\uparrow
two poles
 $\oplus s=p_2$

$k_1 \rightarrow$ no change, do cover up method
for $k_2 \oplus k_3$ either { (1) equating coefficients
(2) differentiating method

Equating Coefficients

$$F(s) = \frac{32s(s+1)}{(s+2)(s+10)^2} = \frac{k_1}{s+2} + \frac{As+B}{(s+10)^2} = \frac{1}{s+2} + \frac{31s-50}{(s+10)^2}$$

$$k_1 = \frac{32s(s+1)}{(s+10)^2} \Big|_{s=-2} = 1$$

$$= \frac{1}{s+2} + \frac{31}{s+10} + \frac{-360}{(s+10)^2}$$

for $A \neq B$ multiply both sides by full original denominator & equate coefficients of s

$$32s(s+1) = k_1(s+10)^2 + (As+B)(s+2)$$

$$32s^2 + 32s = s^2 + 20s + 100 + As^2 + 2As + Bs + 2B$$

$$f(t) = [e^{-2t} + 31e^{-10t} - 360te^{-10t}] u(t)$$

$$\begin{cases} s^2: & 32 = 1 + A \\ s: & 32 = 20 + 2A + B \\ 1: & 0 = 100 + 2B \end{cases} \rightarrow \begin{array}{l} A = 31 \\ B = -50 \end{array}$$

