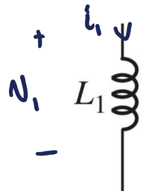
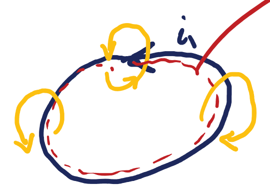
Inductance: Review

We know behavior of an inductor

$$v_1 = L\frac{di_1}{dt}$$



More fundamentally



magnetic flux

$$\Phi_S = \alpha_1 i_1$$

(from Ampere's Law)

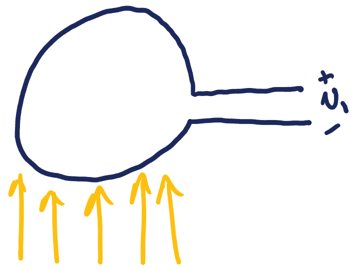
α_1 = some geometric constant

If these are the same loop

$$\Phi_S = \Phi_E$$

$$v_1 = \frac{d}{dt}\alpha_1 i_1$$

$$v_1 = \alpha_1 \frac{di_1}{dt}$$



from Faraday's law

$$|v_1| = \frac{d\Phi_E}{dt}$$

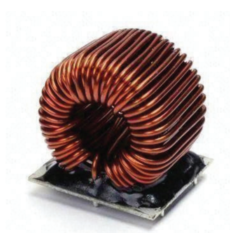
for an N-turn coil

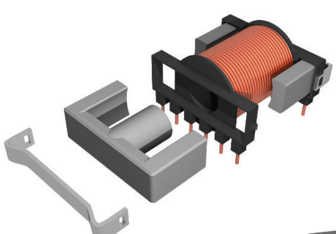
$$\Phi_S = N\alpha_1 i_1$$

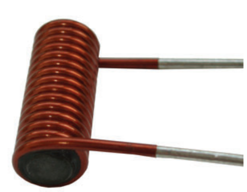
$$|v_1| = N\frac{d\Phi_E}{dt}$$

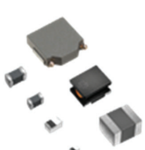
$$v_1 = N^2\alpha_1\frac{di_1}{dt}$$

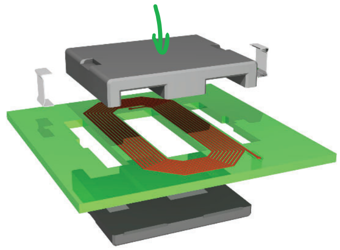
Some Inductor Examples







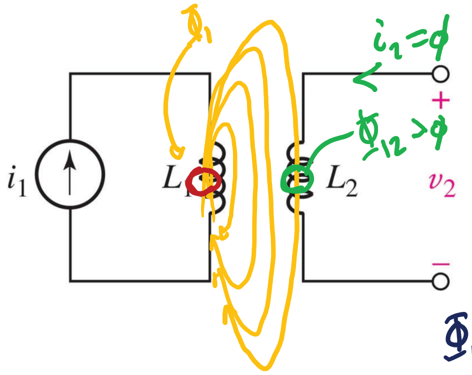




high μ_r material (ferrite)

Magnetic Coupling

→ occurs when (intentionally or not) flux from one inductance impinges on another



Case 1: $i_2 = 0$

$\Phi_1 = \alpha_1 N_1 i_1$ $\quad v_1 = N_1 \frac{d\Phi_1}{dt} \rightarrow v_1 = \underbrace{N_1^2 \alpha_1}_{L_1} \frac{di_1}{dt}$

Looking at L_2:

Φ_{12} = portion of Φ_1 that impinges on L_2's windings

$|v_2| = N_2 \frac{d\Phi_{12}}{dt}$, $\phi \leq \Phi_{12} < \Phi_1$

Let's say $\Phi_{12} = \alpha_{12} N_1 i_1$

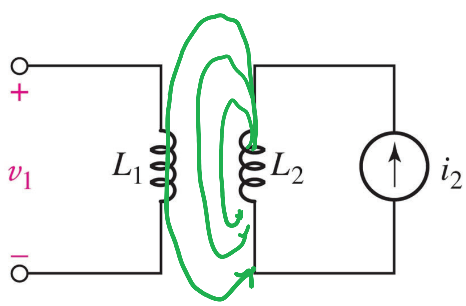
$\Big\}\ v_2 = \underbrace{\alpha_{12} N_2 N_1}_{= M} \frac{di_1}{dt}$

$= M =$ "Mutual inductance"

when $i_2 = \phi$

$$\begin{cases} v_1 = L_1 \frac{di_1}{dt} \\ v_2 = M \frac{di_1}{dt} \end{cases}$$

Magnetic Coupling



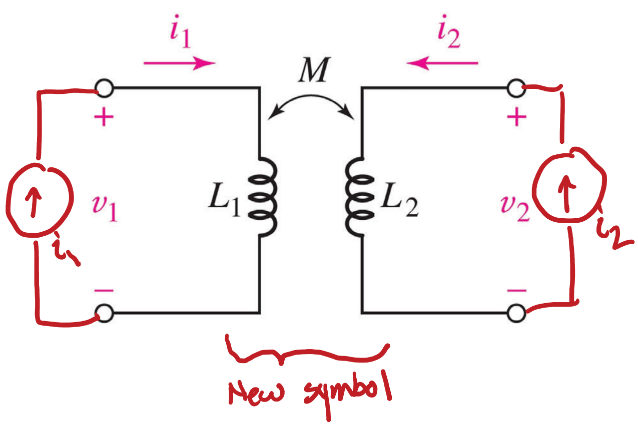
Case 2: $i_1 = \phi$

same circuit as previous, just mirrored & relabeled

when $i_1 = \phi$

$$\begin{cases} v_2 = L_2 \frac{di_2}{dt} \\ v_1 = M \frac{di_2}{dt} \end{cases}$$

Mutual Inductance



Apply superposition:

when $i_2 = 0$

$$\begin{cases} v_1 = L_1 \frac{di_1}{dt} \\ v_2 = M \frac{di_1}{dt} \end{cases}$$

when $i_1 = 0$

$$\begin{cases} v_1 = M \frac{di_2}{dt} \\ v_2 = L_2 \frac{di_2}{dt} \end{cases}$$

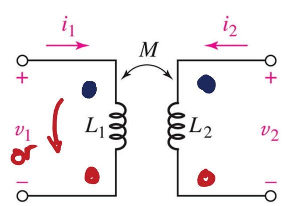
$$v_1 = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

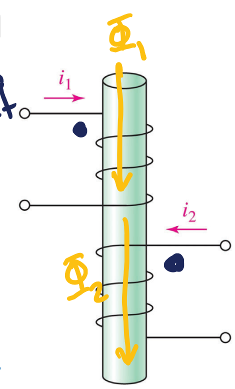
Sign depends on winding arrangement

Symbols and Dot Convention

Note: Always assign passive sign convention

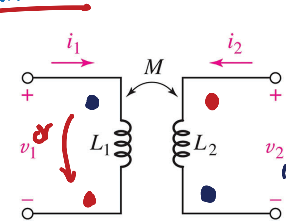


$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

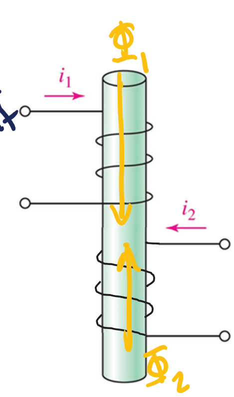


Physically:

If both winding currents enter the dotted terminals, the fluxes are additive



$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



Circuit:

Current flowing into the dotted terminal of one winding will produce an open-circuit voltage that is positive at the dotted terminal of the other