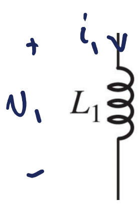
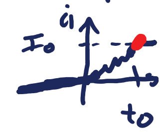
Energy Storage

prior to $t=\phi$, $i_1=\phi$, @ t_0, $I_0 = i_1$

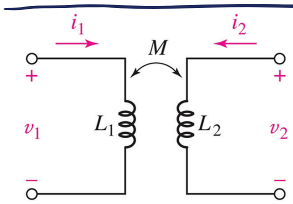




$$E_L = \int_0^{t_0} p_L(t)\,dt = \int_0^{t_0} v_1(t)\, i_1(t)\,dt = \int_0^{t_0} \underline{L_1 \underbrace{\frac{di_1}{dt} i_1(t)}}\,dt$$

$$E_L = L_1 \int_0^{t_0} \left[\frac{d}{dt} i_1(t)^2\right] \cdot \frac{1}{2}\,dt = \frac{1}{2} L_1 \left[i_1(t)^2\right]\Big|_{i_1(t=0)}^{i_1(t=t_0)} = \frac{1}{2} L_1 I_0^2$$

$$\boxed{E_L = \frac{1}{2} L_1 I_0^2}$$



$$E_{12} = \int_0^{t_0} \left(i_1(t)\cdot v_1(t) + i_2(t)\cdot v_2(t)\right) dt$$

before $t=0$ $i_1(t)=0$ & $i_2(t)=0$
@ t_0 $i_1(t) = I_{01}$, $i_2(t) = I_{02}$

$$E_{12} = \int_0^{t_0} \left(i_1\left(L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}\right) + i_2\left(L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}\right)\right) dt$$

$$E_{12} = \int_0^{t_0} \underline{L_1 \frac{di_1}{dt} i_1} + L_2 \frac{di_2}{dt} i_2 \pm M\left(i_1 \frac{di_2}{dt} + i_2 \frac{di_1}{dt}\right) dt$$

$$E_{12} = \frac{1}{2} L_1 I_{01}^2 + \frac{1}{2} L_2 I_{02}^2 \pm M \int_0^{t_0} \frac{d}{dt}(i_1 \cdot i_2)\,dt$$

$$\boxed{E_{12} = \frac{1}{2} L_1 I_{01}^2 + \frac{1}{2} L_2 I_{02}^2 \pm M I_{01} I_{02}}$$

$$E_{12} = \frac{1}{2} L_1 I_{01}^2 + \frac{1}{2} L_2 I_{02}^2 \pm M I_{01} I_{02} \geq \phi$$

pick negative sign for "minimum" $E_{12} \geq \phi$

$$M \leq \frac{\frac{1}{2} L_1 I_{01}^2 + \frac{1}{2} L_2 I_{02}^2}{I_{01} I_{02}} = \frac{1}{2} L_1 \frac{I_{01}}{I_{02}} + \frac{1}{2} L_2 \frac{I_{02}}{I_{01}}$$

let $x = \frac{I_{01}}{I_{02}}$ $\longrightarrow$ $M \leq \underline{\frac{1}{2} L_1 x + \frac{1}{2} L_2 \frac{1}{x}}$

find max & min w.r.t. $x = \frac{I_{01}}{I_{02}}$ & make sure inequality holds

$$\frac{dM}{dx} = \frac{1}{2} L_1 + \frac{1}{2} L_2 \frac{-1}{x^2} = \phi \rightarrow x = \sqrt{L_2/L_1}$$

$$\frac{\partial^2 M}{\partial x^2} = \frac{1}{2} L_2 \frac{+2}{x^3} = \frac{L_2}{x^3} > \phi$$

$E_{12} \geq \phi$ must hold @ minimum at $x = \sqrt{L_2/L_1}$

$$M \leq \frac{1}{2} L_1 \sqrt{L_2/L_1} + \frac{1}{2} L_2 \sqrt{L_1/L_2}$$

$$\boxed{M \leq \sqrt{L_1 L_2}}$$

Coupling Coefficient

Define $\boxed{k = \frac{M}{\sqrt{L_1 L_2}}} \equiv$ "coupling coefficient"

Always $0 \le |k| \le 1$

$k = 0 \rightarrow$ two seperate inductors

coupled inductors

$k = 1 \rightarrow$ perfect coupling

$\rightarrow$ "transformer"

Transformers

Special case where $\underline{k = 1} \iff M = \sqrt{L_1 L_2}$

$$\begin{cases} v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{cases} \xRightarrow{k=1} \begin{cases} v_1 = L_1 \frac{di_1}{dt} + \sqrt{L_1 L_2} \frac{di_2}{dt} \\ v_2 = L_2 \frac{di_2}{dt} + \sqrt{L_1 L_2} \frac{di_1}{dt} \end{cases}$$

assume positive coupling

$$v_2 = v_1 \sqrt{\frac{L_2}{L_1}}$$

$$v_2 = v_1 \sqrt{\frac{\cancel{\alpha_2} N_2^2}{\cancel{\alpha_1} N_1^2}}$$

if $k = 1$, $\alpha_1 = \alpha_2$

$$v_2 = v_1 \frac{N_2}{N_1} \rightarrow \text{"turns ratio"}$$

$$\boxed{\frac{v_2}{N_2} = \frac{v_1}{N_1}}$$

New circuit symbol

where $N = \frac{N_2}{N_1}$