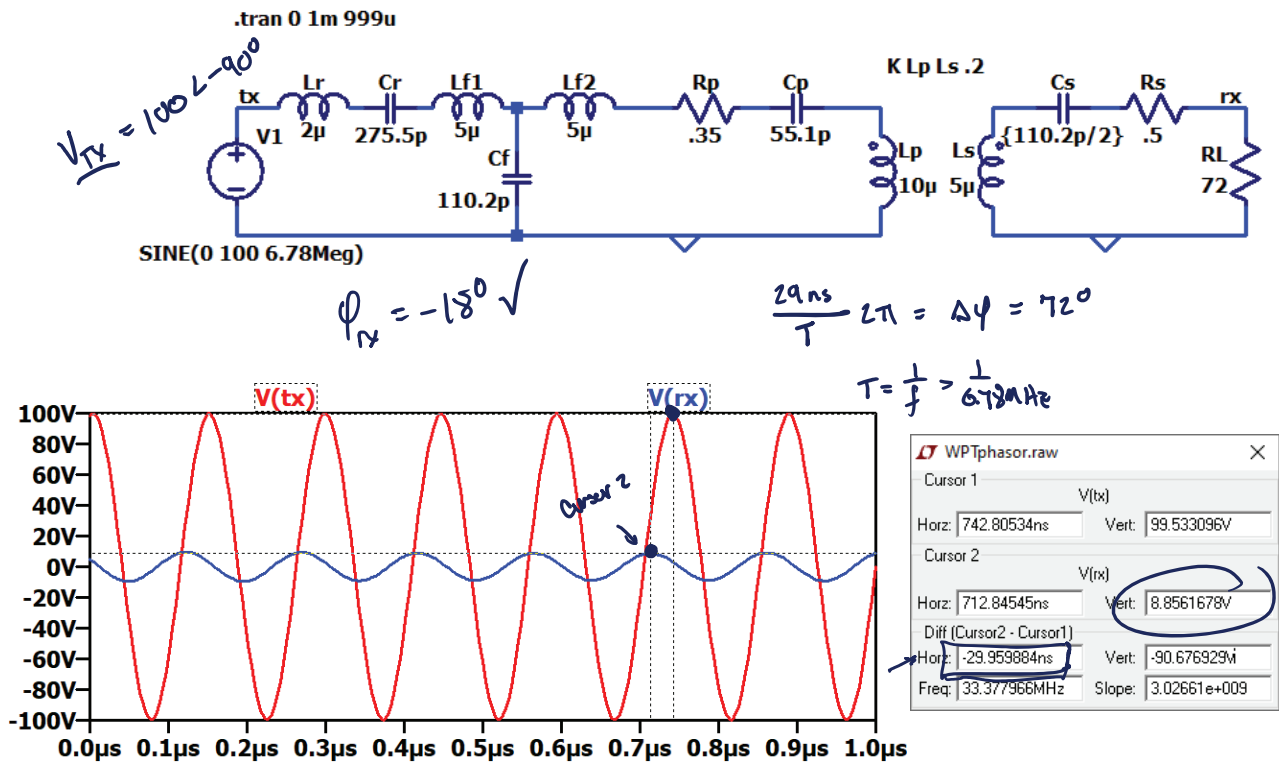
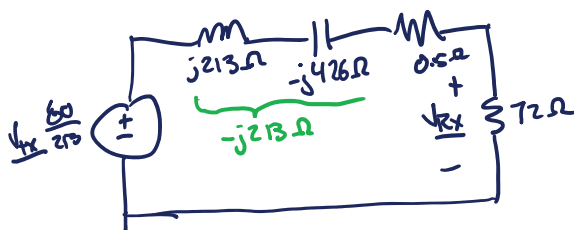


Circuit Simulation 2



from last class analysis



$$\underline{V_{Rx}} = \underline{V_{tx}} \frac{60}{213} \frac{72}{72.5 + (-j213)}$$

where $\underline{V_{tx}} = 100 \angle -90^\circ = 100 e^{j(-\frac{\pi}{2})} = -j100$

$$\textcircled{1} \underline{V_{Rx}} = (-j100) \frac{60}{213} \frac{72}{72.5 - j213} \frac{(72.5 + j213)}{(72.5 + j213)} = -j \frac{6000}{213} \frac{72(72.5) + j72(213)}{72.5^2 + 213^2}$$

$$= \left(\frac{6000}{213} \frac{72(213)}{72.5^2 + 213^2} \right) - j \left(\frac{6000}{213} \frac{72(72.5)}{72.5^2 + 213^2} \right) = 8.53 - j2.905$$

$$\underline{V_{Rx}} = \sqrt{8.53^2 + 2.905^2} e^{j \tan^{-1} \left(\frac{-2.905}{8.53} \right)} = 9 \angle -18.8^\circ$$

$$\textcircled{2} \underline{V_{Rx}} = 100 e^{j(-\frac{\pi}{2})} \frac{60}{213} \cdot 72 \left[\sqrt{72.5^2 + 213^2} e^{j \tan^{-1} \left(\frac{-213}{72.5} \right)} \right]^{-1}$$

$$\underline{V_{Rx}} = \left(100 \frac{60}{213} \cdot \frac{72}{\sqrt{72.5^2 + 213^2}} \right) e^{j \left(-\frac{\pi}{2} - \tan^{-1} \left(\frac{-213}{72.5} \right) \right)} = 9 \angle -18.8^\circ$$

$V_{Rx}(t) = 9 \cos(\omega t - 18.8^\circ)$

Chapter 11

AC CIRCUIT POWER ANALYSIS

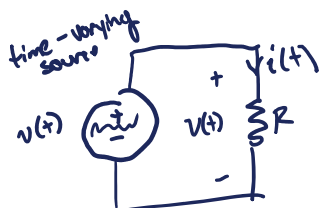
Average Power



$$P = V \cdot I, \quad V = IR$$

for resistors

$$P = \frac{V^2}{R} = I^2 R$$



$$p(t) = v(t) \cdot i(t)$$

↳ instantaneous power

$$p(t) = i(t)^2 R = \frac{v(t)^2}{R}$$

Average power over some time interval $[t_1, t_1 + T]$

$$P = \frac{1}{T} \int_{t_1}^{t_1+T} p(t) dt$$

Average power over all time

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t) dt$$

For any periodic signals, the average over a positive integer number of periods is the average for all time.

Power in a Resistor

Assume time-varying bias to a resistor w/ periodic voltage / current with period T

$$P_R = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \int_0^T i(t)^2 R dt = R \frac{1}{T} \int_0^T i(t)^2 dt$$

Let's make this expression look like $P = I^2 R$

$$P_R = \left[\sqrt{\frac{1}{T} \int_0^T i(t)^2 dt} \right]^2 R$$

↳ square root of the average of $i(t)^2$

↳ root-mean-squared (rms) current

Define: $I_{rms} = \sqrt{\frac{1}{T} \int_0^T i(t)^2 dt}$ $P = I_{rms}^2 R$

↳ Book calls this "effective" current I_{eff}

RMS of a sinusoid

$$i(t) = I_A \cos(\omega t + \phi_I)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$i_{rms} = \sqrt{\frac{1}{T} \int_0^T [I_A \cos(\omega t + \phi_I)]^2 dt}$$

Trig identity
 $\cos^2 \theta = \frac{1}{2} [1 + \cos 2\theta]$

$$i_{rms}^2 = I_A^2 \frac{1}{T} \int_0^T \left[\frac{1}{2} + \frac{1}{2} \cos(2\omega t + 2\phi_I) \right] dt$$

$$= I_A^2 \frac{1}{T} \left[\frac{1}{2} t - \frac{1}{4\omega} \sin(2\omega t + 2\phi_I) \right] \Big|_0^{\frac{2\pi}{\omega}}$$

$$= I_A^2 \frac{\cancel{2\pi}}{\cancel{2\pi}} \left[\frac{\cancel{\pi}}{\cancel{\omega}} - \phi \right]$$

$$i_{rms}^2 = \frac{I_A^2}{2}$$

$$\rightarrow \boxed{i_{rms} = \frac{I_A}{\sqrt{2}}}$$