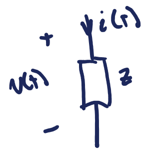
Power with Sinusoidal Sources



In steady-state with $v(t)$ & $i(t)$ as single-frequency sinusoids

$$\underline{V} = \underline{I}Z \rightarrow V_A e^{j\varphi_V} = I_A e^{j\varphi_I} |Z| e^{j\varphi_Z}$$

$$\underline{V} = I_A |Z| e^{j(\varphi_I + \varphi_Z)}$$

$$\begin{cases} V_A = I_A |Z| \\ \varphi_V = \varphi_I + \varphi_Z \end{cases}$$

For power, we <u>cannot</u> just multiply $\underline{V}$ & $\underline{I}$

$$p(t) = v(t)i(t) = \underline{Re}\{V_A e^{j\varphi_Z} \underline{e^{j\omega t}}\} \cdot \underline{Re}\{I_A e^{j\varphi_I} \underline{e^{j\omega t}}\}$$

→ $Re\{\cdot\}$ & $e^{j\omega t}$ matter

Go back to the time domain:

Trig Identity

$$2\cos\theta\cos\varphi = \cos(\theta+\varphi) + \cos(\theta-\varphi)$$

$$p(t) = V_A \cos(\omega t + \varphi_V) \cdot I_A \cos(\omega t + \varphi_I)$$

$$= V_A I_A \cos(\omega t + \varphi_V)\cos(\omega t + \varphi_I)$$

$$= \underbrace{\frac{V_A I_A}{2}\cos(2\omega t + \varphi_V + \varphi_I)}_{\text{sinusoid @ } 2\omega \text{ AC}} + \underbrace{\frac{V_A I_A}{2}\cos(\underbrace{\varphi_V - \varphi_I}_{=\varphi_Z})}_{\text{No time-variation DC}}$$

In $\varphi_V - \varphi_I$, order doesn't matter

Average Power $\quad P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{V_A I_A}{2}\cos(\varphi_V - \varphi_I) = V_{rms} I_{rms} \cos(\varphi_V - \varphi_I)$

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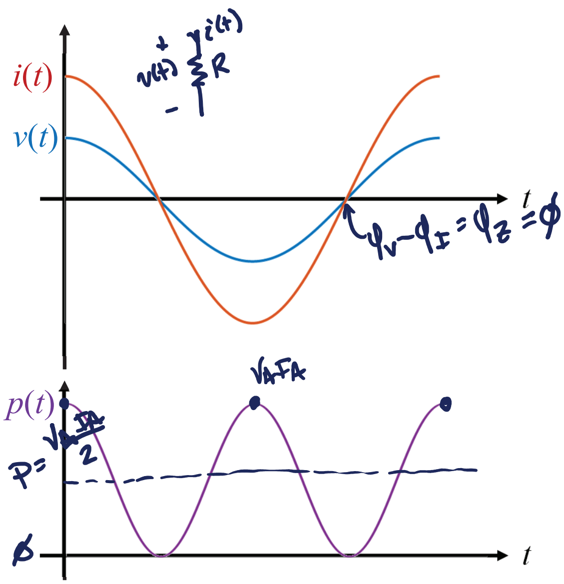
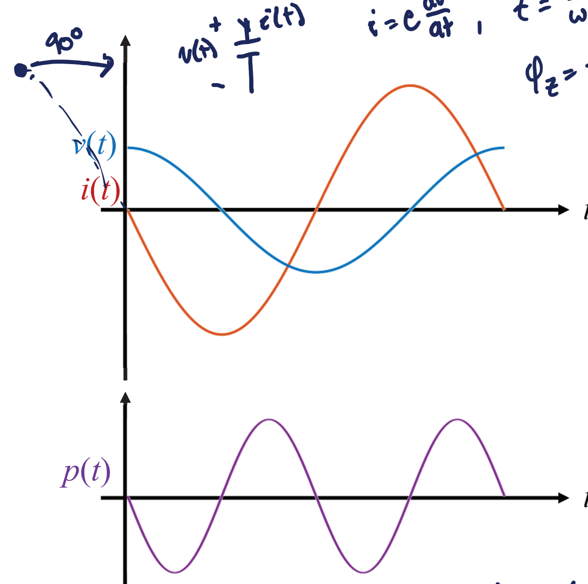


$$P_R = \frac{V_A I_A}{2} = V_{rms} I_{rms}$$

$i = C\frac{dv}{dt}$, $Z = \frac{-j}{\omega C}$

$\varphi_Z = -90°$



$$\cos(\varphi_V - \varphi_I) = \cos(90°) = 0$$

so

$$P = 0$$