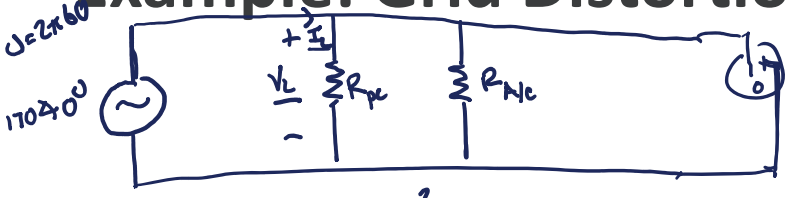
Example: Grid Distortion



PC: about 1kW } power consumption
A/c: 3.5 kW

what are R_{pc} & $R_{A/c}$?

Ohm's law for impedances

$\underline{V} = \underline{I}\,\underline{Z}$

for resistor $\underline{I} = \frac{\underline{V}}{R}$

$\phi_Z = 0$ for resistor

$$P_R = \frac{|V_R||I_R|}{2}\cos(\phi_V - \phi_I) = \frac{|V_R||I_R|}{2}$$

$$P_R = \frac{|V_R|^2}{2R} \quad \text{then} \quad |I_R| = \frac{|V_R|}{R}$$

$$R_{pc} = \frac{|V_L|^2}{2P_{pc}} = \frac{170^2}{2\cdot 1kW} = 14.5\,\Omega$$

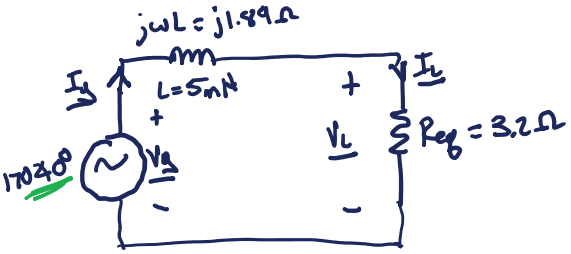
$$R_{A/c} = \frac{170^2}{2\cdot 3.5kW} = 4.1\,\Omega$$

$R_{eq} = R_{pc} \| R_{A/c} = 3.2\,\Omega$

$$\underline{I_L} = \frac{\underline{V_g}}{R_{eq}} = 53\angle 0^\circ\ A$$

$$\underline{S_g} = \frac{1}{2}\underline{V_g}\,\underline{I_L}^* = \frac{1}{2}(170e^{j0})(53e^{j0}) = 4.5kW + 0j\ VAR$$

$$PF = \frac{P}{|S|} = 1$$



$$\underline{I_g} = \underline{I_L} = \frac{\underline{V_g}}{R_{eq} + j\omega L} = 45\angle -30^\circ$$

$$\underline{V_L} = \underline{I_g} R_{eq} = 146.5\angle -30^\circ$$

$$P_L = \frac{|V_L||I_L|}{2}\cos(0^\circ) = 3.3kW$$

In a more realistic model R_{eq} might change to keep the same power
– Neglect for now.

Complex power from grid

$$S_g = \frac{1}{2}\underline{V_g}\,\underline{I_g}^* = \frac{1}{2}(170\angle 0^\circ)(45\angle 30^\circ) = 3.8kW\angle 30^\circ$$

$S = 3.3kW + j1.9kW$ (P, Q)

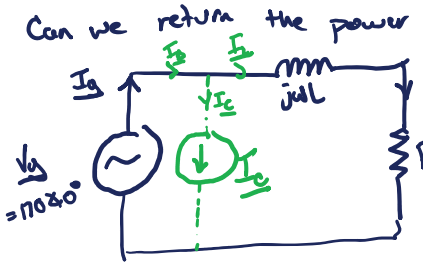
$$PF = \frac{P}{|S|} = 0.87 \quad \text{lagging}$$

could also use

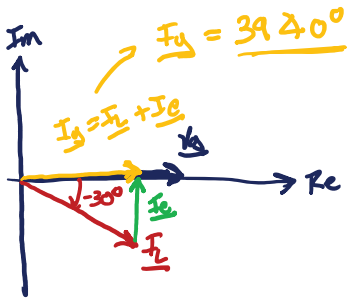
$PF = \cos(\phi_V - \phi_I)$

Defined for current relative to voltage
"lagging" indicates our load is inductive, "leading" capacitive

Can we return the power factor to unity



$I_R = 45 \angle -30°$



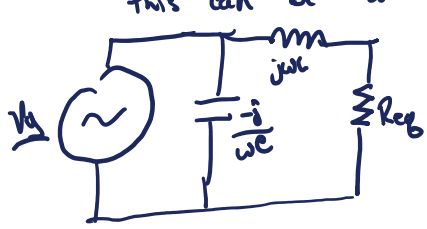
Try to add I_c such that phase of $I_g = 0°$

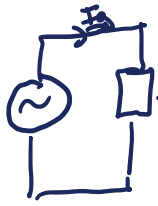
$$I_c = 45\sin(30°) \angle 90°$$
$$= 22.5 \angle 90° = j22.5$$

$$\frac{V_g}{I_c} = Z = \frac{110e^{j0}}{22.5e^{j\pi/2}} = \frac{110}{22.5} e^{-j\frac{\pi}{2}} = 7.56 \angle -90° = \frac{-j}{1/7.56} \Rightarrow \frac{-j}{\omega C}$$

$$C = \frac{1}{\omega 7.56} = 350\,\mu F$$

this can be a capacitor with





$Z_{eq} = \frac{-j}{\omega C} \| (jwL + R_{eq}) = 4.3 + 0j\ \Omega$ transformed the impedance

$S = V_g I_g^* = \frac{|V_g||I_g|}{2} \cos(\phi_v - \phi_i)$

$= 3.3kW + 0j$

PF = 1