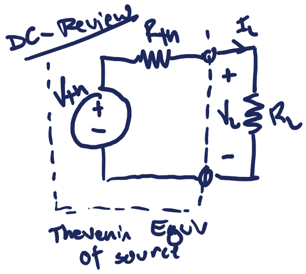
Maximum Power Transfer



What resistance R_L results in maximum $P_L = I_L V_L$?

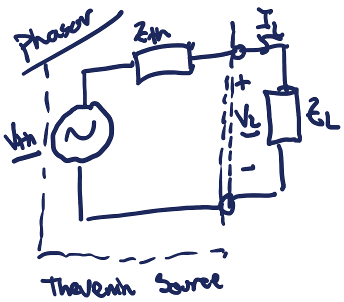
$$P_L = V_{th} \frac{R_L}{R_L + R_{th}} \cdot \frac{V_{th}}{R_L + R_{th}} = V_{th}^2 \frac{R_L}{(R_L + R_{th})^2}$$

$$\frac{\partial P_L}{\partial R_L} = \emptyset = V_{th}^2 \left[\frac{1}{(R_L + R_{th})^2} + \frac{-2 R_L}{(R_L + R_{th})^3} \right]$$

$$R_L = \frac{1}{2}(R_L + R_{th}) \longrightarrow \boxed{R_L = R_{th}}$$

Note: maximizing power may not always be a good idea.

Efficiency $\frac{P_{out}}{P_{in}} = \eta = \frac{I_L^2 R_L}{I_L^2 R_{th} + I_L^2 R_L} = \frac{R_L}{R_L + R_{th}} \longrightarrow$ 50% at maximum power

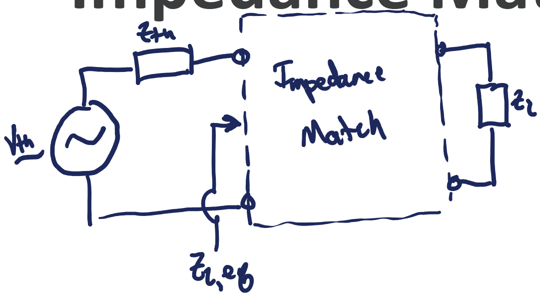


What value of Z_L maximizes real power to the load?

If we set $Z_L = Re\{Z_{th}\} - j\, Im\{Z_{th}\}$

$$\boxed{Z_L = Z_{th}^*}$$

Impedance Matching



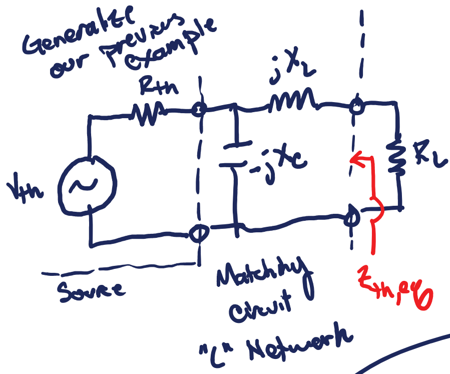
Signals → RF → $Z_{eq} = Z_{th}^*$

Signals → comm → $Z_{L,eq} = Z_{th}$

power converters → $R_{L,eq} \to \infty \gg R_{th}$

grid applications → periodically correct PF

Example Matching Circuits



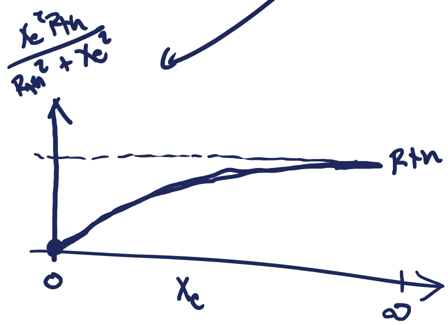
$$Z_{th,eq} = R_{th} \| (-jX_C) + jX_L$$

$$= \frac{-jX_C R_{th}}{R_{th} - jX_C} + jX_L$$

$$= \left[\frac{X_C^2 R_{th}}{R_{th}^2 + X_C^2}\right] + j\left[\frac{-X_C R_{th}^2}{R_{th}^2 + X_C^2}\right] + jX_L$$

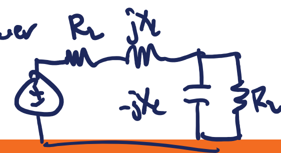
for max power transfer — set to R_L

cancel reactance to zero



→ works if $R_{th} > R_L$ → this configuration lowers source resistance

to lower R_L



If I swap L & C? same result