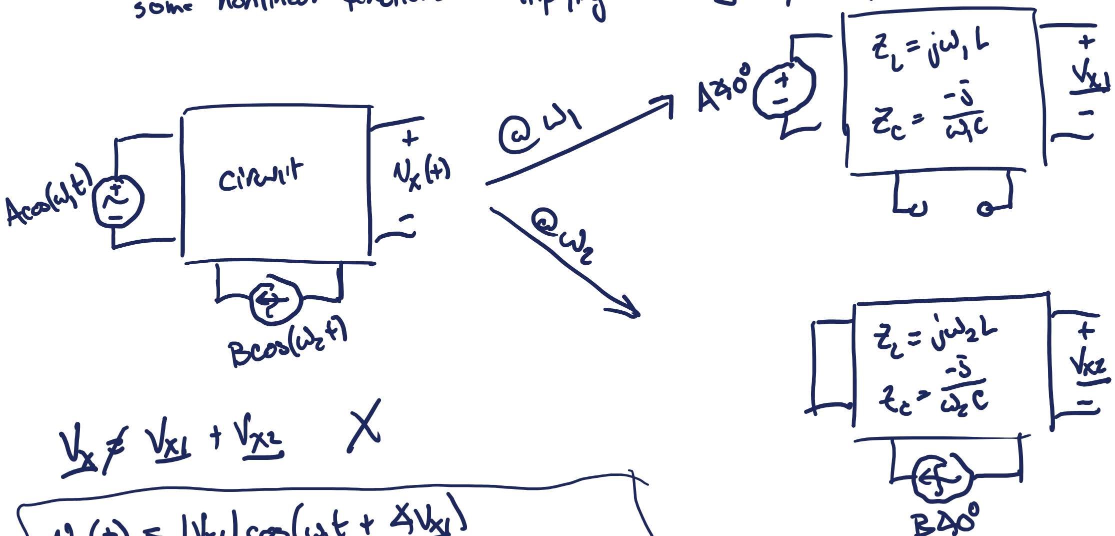
Phasor Superposition → Problem 10-63 on HW #4

- Phasor Transformation valid only for a single frequency
- Superposition can be applied if we comput the transform at each frequency.

superposition is guaranteed to work for linear functions → $\left.\begin{array}{l} f(x_1)=y_1 \\ f(x_2)=y_2 \end{array}\right\} f(x_1+x_2)=y_1+y_2$

Linear functions include $+/-$, $\frac{d}{dt}$, & multiplication by constant

some nonlinear functions: multiplying two signals, $(x)^2$, ...



$V_x \neq V_{x1} + V_{x2}$ X

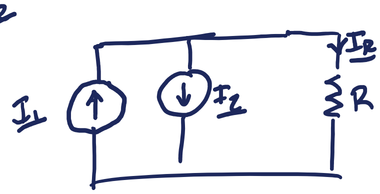
$$\boxed{v_x(t) = |V_{x1}|\cos(\omega_1 t + \angle V_{x1}) + |V_{x2}|\cos(\omega_2 t + \angle V_{x2})}$$

Power Spectrum

End of 11.4

Power calculations are nonlinear → superposition not guaranteed.

example



If $I_1 = I_2$

by inspection $I_R = \emptyset$ so $P_R = \emptyset$

But if we (incorrectly) apply superposition

$$\left.P_1 = \frac{|I_1|^2 R}{2}, \quad P_2 = \frac{|I_2|^2 R}{2}\right\} X$$

$$P_1 + P_2 = |I_1|^2 R$$

But, superposition of powers does work if all sources are at different frequency.

$$p_R(t) = i_R(t)^2 R = \left[|I_1|\cos(\omega_1 t) - |I_2|\cos(\omega_2 t)\right]^2 R$$

average $$P_R = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left[|I_1|\cos(\omega_1 t) - |I_2|\cos(\omega_2 t)\right]^2 R\, dt$$

$$= \lim_{T\to\infty} \frac{R}{T}\int_{-T/2}^{T/2} \underbrace{|I_1|^2\cos^2(\omega_1 t)}_{P_1} + \underbrace{|I_2|^2\cos^2(\omega_2 t)}_{P_2} - \underbrace{2|I_1||I_2|\cos(\omega_1 t)\cos(\omega_2 t)}_{2|I_1||I_2|\frac{1}{2}\left(\cos(\omega_1 t+\omega_2 t) + \cos(\omega_1 t - \omega_2 t)\right)}\, dt$$

Average $= \emptyset$ if $\omega_1 \neq \omega_2$

Limitations of Phasor Analysis

① Single frequency
② Only steady-state response
③ sinusoidal inputs & waveforms only

} Developed for particular solutions to ODE
Pattern matching & know that for sinusoids in, we get sinusoids out, only mag & phase change.

Want to develop analysis techniques w/out these limitations

① superposition can help
frequency response → valid for all ω (chapter 15)

③ Fourier series & transform can express arbitrary* waveforms as a sum of sinusoids
Ch 17

② Homogeneous solution needed
Need exponentials → Ch 14, Laplace Transforms

Frequency Response

solve with ω as a variable

$$\underline{V_o} = \underline{V_I}\,\frac{\frac{-j}{\omega C}}{R - \frac{j}{\omega C}} = \underline{V_I}\,\boxed{\frac{1}{jR\omega C + 1}}$$

↓ some complex number
$H(j\omega)$
↓
Frequency Response