Complex Form of Fourier Series

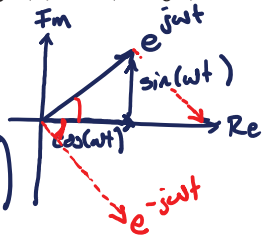
for signal periodic about $T = \frac{2\pi}{\omega_0}$

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)$$

$$A_0 = \frac{1}{T}\int_0^T f(t)dt \qquad A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt \qquad B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$$

Euler: $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \rightarrow \cos(\omega t) = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)$

$$\sin(\omega t) = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

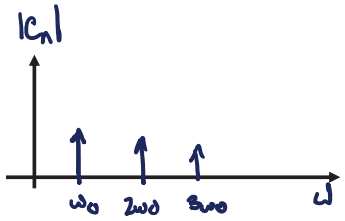


$$f(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2}\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right) + \frac{B_n}{2j}\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right)$$

$$= A_0 + \sum_{n=1}^{\infty} e^{jn\omega_0 t}\left(\frac{A_n - jB_n}{2}\right) + e^{-jn\omega_0 t}\left(\frac{A_n + jB_n}{2}\right)$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

$$\boxed{C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t}\,dt}$$



Non-periodic Waveforms: Fourier Transform

Non-periodic waveform is "say" it repeats after period $T \rightarrow \infty$

$\omega_0 = \frac{2\pi}{T} \rightarrow 0$

Periodic: $C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\,dt$

Non-periodic: $TC_n = \boxed{\int_{-\infty}^{\infty} f(t) e^{-j\omega t}\,dt = F(\omega)}$ } Fourier Transform

Periodic: $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

Non-periodic $\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t}\,d\omega}$