Inverse Transforms

if $F(s)$ is $\mathcal{L}\{f(t)\}$, $\mathcal{L}^{-1}\{F(s)\} = f(t)$ (uniqueness)

Often we're interested in $\mathcal{L}^{-1}\{\ \}$ of signals that look like a ratio of polynomials

$$F(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}$$

if $m \leq n$ → physically realizable system (causal system)

often, we'll need to factor the polynomials

ex/ $F(s) = \frac{10}{s^2+4s+4} = \frac{10}{(s+2)^2}$

$$\mathcal{L}^{-1}\left\{\frac{10}{(s+2)^2}\right\} = 10\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\} = \boxed{10te^{-2t}u(t) = f(t)}$$

ex/ $F(s) = \frac{5s+1}{s+1}$ here $m \geq n$, so use long division

$$\begin{array}{r} 5 \\ s+1 \overline{)\,5s+1} \\ -(5s+5) \\ \hline -4 \end{array}$$

$$\frac{5s+1}{s+1} = 5 - \frac{4}{s+1}$$

$$\mathcal{L}^{-1}\left\{5 - \frac{4}{s+1}\right\} = 5\mathcal{L}^{-1}\{1\} - 4\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = \boxed{5\delta(t) - 4e^{-t}u(t) = f(t)}$$

Partial Fraction Expansion / Decomposition

$$\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \cdots \frac{k_n}{s-p_n}$$

k_i are called "residuals"

where all p_i are unique & $n > m$

Find k_i by multiplying both sides by $(s-p_i)$ & evaluating at $s=p_i$

e.g. @ $i=2$

$$\cancel{(s-p_2)}\frac{(s-z_1)(s-z_2)-(s-z_m)}{(s-p_1)\cancel{(s-p_2)}\cdots(s-p_n)} = \cancel{\frac{k_1(s-p_2)}{s-p_1}} + \frac{k_2\cancel{(s-p_2)}}{\cancel{s-p_2}} + \cdots \cancel{\frac{k_n(s-p_2)}{s-p_n}} = k_2$$

ex/ $F(s) = \frac{4(s+2)}{s^2+4s+3} = \frac{4(s+2)}{(s+1)(s+3)} = \frac{k_1}{s+1} + \frac{k_2}{s+3} = \frac{2}{s+1} + \frac{2}{s+3}$

for k_1 ⟹ $k_1 = \left.\frac{4(s+2)}{s+3}\right|_{s=-1} = 2$

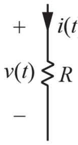
for k_2 ⟹ $k_2 = \left.\frac{4(s+2)}{s+1}\right|_{s=-3} = 2$

$$f(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = 2\left(e^{-t} + e^{-3t}\right)u(t)$$

if p_i are not unique, polynomial residual e.g. $\frac{k_1 s + k_2}{(s-p_i)^2}$

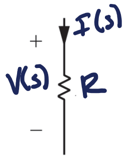
Circuit Laplace Transform

Time Domain | **Phasor Domain** | ***s*-Domain**



$v(t) = i(t)R$

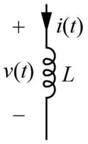
$V = IR$



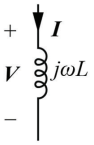
$\mathcal{L}\{v(t)\} = \mathcal{L}\{i(t)R\}$

$\boxed{V(s) = I(s)R}$

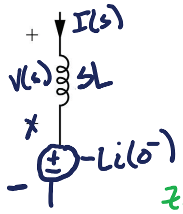
$Z_R = R$ still use "Impedance"



$v(t) = L\frac{di}{dt}$



$V = j\omega L I$



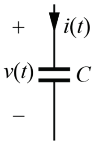
$\mathcal{L}\{v(t)\} = \mathcal{L}\{L\frac{di}{dt}\}$

$V(s) = L[sI(s) - i(0^-)]$

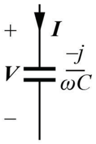
$\boxed{V(s) = sLI(s) - Li(0^-)}$

$\boxed{I(s) = \frac{1}{sL}V(s) + \frac{1}{s}i(0^-)}$

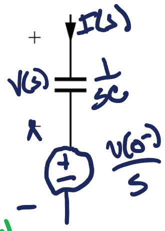
$Z_L = sL$



$i(t) = C\frac{dv}{dt}$



$V = \frac{-j}{\omega C}I$



$I(s) = C[sV(s) - v(0^-)]$

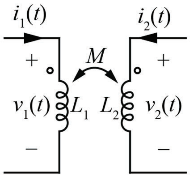
$\boxed{I(s) = sCV(s) - Cv(0^-)}$

$\boxed{V(s) = \frac{1}{sC}I(s) + \frac{v(0^-)}{s}}$

IC = 0 & $s = j\omega$

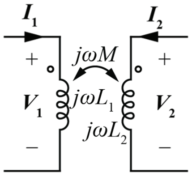
$Z_C = \frac{1}{sC}$

Time Domain | **Phasor Domain** | ***s*-Domain**



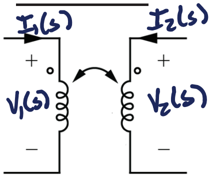
$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$



$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$



$V_1(s) = L_1[sI_1(s) - i_1(0^-)] + M[sI_2(s) - i_2(0^-)]$

$V_2(s) = M[sI_1(s) - i_1(0^-)] + L_2[sI_2(s) - i_2(0^-)]$

Alternately replace w/ equivalent circuit first.

Ideal XFs (ie $k=1$ & L's $\to \infty$)

No change

$\frac{V_1(s)}{N_1} = \frac{V_2(s)}{N_2}$

$N_1 I_1(s) + N_2 I_2(s) = 0$