Repeated Roots – Equating Coefficients

ex/ $F(s) = \dfrac{32s(s+1)}{(s+2)(s+10)^2} = \dfrac{k_1}{s+2} + \dfrac{k_x s + k_y}{(s+10)^2}$

for k_1, find the same way $\quad k_1 = \dfrac{32s(s+1)}{(s+10)^2}\Bigg|_{s=-2} = \boxed{1 = k_1}$

find k_x \$ k_y $\rightarrow$ multiply both sides by the full denominator of original $F(s)$

$$32s(s+1) = k_1(s+10)^2 + (k_x s + k_y)(s+2)$$

$$32s^2 + 32s = s^2(k_1 + k_x) + s(20k_1 + 2k_x + k_y) + 100 + 2k_y$$

equating coefficients

s^2: $32 = k_1 + k_x \rightarrow k_x = 31$

s: $32 = 20k_1 + 2k_x + k_y$ ✓

1: $0 = 100 + 2k_y \rightarrow k_y = -50$

$$\boxed{F(s) = \frac{32s(s+1)}{(s+2)(s+10)^2} = \frac{1}{s+2} + \frac{31s - 50}{(s+10)^2}}$$

$$\boxed{= \frac{1}{s+2} + \frac{31}{s+10} + \frac{-360}{(s+10)^2}}$$

Repeated Roots - Differentiating

ex/ $F(s) = \dfrac{32s(s+1)}{(s+2)(s+10)^2} = \dfrac{k_1}{s+2} + \dfrac{k_2}{s+10} + \dfrac{k_3}{(s+10)^2}$

for k_1 \$ k_3 find as normal

$k_1 = \dfrac{32s(s+1)}{(s+10)^2}\Bigg|_{s=-2} = \boxed{1 = k_1}$

$k_3 = \dfrac{32s(s+1)}{(s+2)}\Bigg|_{s=-10} = \boxed{-360 = k_3}$

for k_2: multiply both sides by $(s+10)^2$ then $\frac{d}{ds}$ before setting $s=-10$

$$\frac{d}{ds}\left[\frac{32s(s+1)}{(s+2)}\right]\Bigg|_{s=-10} = \frac{d}{ds}\left[\cancel{\frac{k_1(s+10)^2}{s+2}}^{\,0} + k_2\cancel{(s+10)}^{\,1} + \cancel{k_3}^{\,0}\right]\Bigg|_{s=-10}$$

$$k_2 = \frac{d}{ds}\left[\frac{32s(s+1)}{(s+2)}\right]\Bigg|_{s=-10} = \frac{(64s+32)(s+2) - (1)(32s^2+32s)}{(s+2)^2}\Bigg|_{s=-10}$$

$$\boxed{k_2 = 31}$$

$$\boxed{F(s) = \frac{1}{s+2} + \frac{31}{s+10} + \frac{-360}{(s+10)^2}}$$

PFE: Complex Roots

ex

$$F(s) = \frac{1}{s^2 - 2s + 2} \qquad \text{roots @ } \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = 1 \pm j$$

↑ Recall that for any real signal/system, complex roots always show up in a conjugate pair

so do residues

$$F(s) = \frac{1}{(s-(1+j))(s-(1-j))}$$

① Do nothing different

$$F(s) = \frac{k_1}{s-(1+j)} + \frac{k_2}{s-(1-j)}$$

$$\begin{cases} k_1 = \left.\frac{1}{s-(1-j)}\right|_{s=1+j} = \frac{1}{2j} = \frac{-j}{2} \\ k_2 = \left.\frac{1}{s-(1+j)}\right|_{s=1-j} = \frac{1}{-2j} = \frac{j}{2} \end{cases}$$

$$F(s) = \frac{-j/2}{s-(1+j)} + \frac{j/2}{s-(1-j)}$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{j}{2}\left[-e^{(1+j)t} + e^{(1-j)t}\right]u(t) = e^{t}\frac{-1}{2j}\left[-e^{jt} + e^{-jt}\right]u(t)$$

$$= e^{t}\,\frac{e^{jt} - e^{-jt}}{2j}\,u(t)$$

$$\boxed{f(t) = e^{t}\sin(t)u(t)}$$