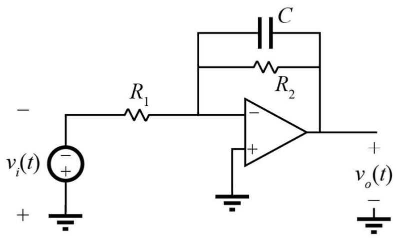
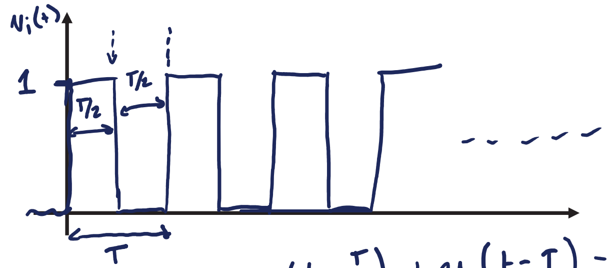
Example Problem





$$v_i(t) = u(t) - u(t-\tfrac{T}{2}) + u(t-T) - u(t-\tfrac{3T}{2}) + \dots$$

$$= \sum_{k=0}^{\infty} u(t-kT) - u(t-kT-\tfrac{T}{2})$$

$$= \sum_{k=0}^{\infty} u(t-k\tfrac{T}{2})(-1)^k$$

$$V_I(s) = \mathcal{L}\{v_i(t)\}$$

$$= \int_{0^-}^{\infty} e^{-st} \sum_{k=0}^{\infty} u(t-k\tfrac{T}{2})(-1)^k$$

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Inverting Gain amp:

Gain $-\frac{Z_2}{Z_1}$ → $\frac{V_o}{V_i} = \frac{Z_2}{Z_1}$

Ideal amp:

(1) (if negative feedback) → virtual short $v_- = v_+$

(2) $i_+ = i_- = \emptyset$

$i_1 = \frac{-v_i}{Z_1}$ $\quad v_o = -i_1 Z_2$

$v_o = -\left(\frac{-v_i}{Z_1}\right) Z_2 = \frac{Z_2}{Z_1}$

$Z_1 = R_1$

$Z_2 = \frac{1}{sC} \| R_2$

$$H(s) = \frac{\frac{R_2/sC}{R_2 + \frac{1}{sC}}}{R_1} = \boxed{\frac{R_2/R_1}{sCR_2 + 1}} = H(s)$$

$$\boxed{H(s) = \frac{R_2}{R_1} \frac{1}{sR_2C + 1}}$$

$$V_I(s) = \sum_{k=0}^{\infty} \int_{0^-}^{\infty} e^{-st} u(t-k\tfrac{T}{2})(-1)^k dt$$

$$V_I(s) = \sum_{k=0}^{\infty} \left[-\frac{1}{s} e^{-st}\right]\Bigg|_{kT/2}^{\infty} (-1)^k$$

$$\boxed{V_I(s) = \sum_{k=0}^{\infty} \frac{1}{s} e^{-sk\frac{T}{2}} (-1)^k}$$

$$V_o(s) = H(s) \cdot V_I(s)$$

$$= \frac{R_2}{R_1} \frac{1}{sR_2C+1} \left(\sum_{k=0}^{\infty} \frac{1}{s} e^{-sk\frac{T}{2}} (-1)^k \right)$$

$$V_o(s) = \sum_{k=0}^{\infty} (-1)^k \frac{R_2}{R_1} e^{-sk\frac{T}{2}} \frac{1}{s} \frac{1}{sR_2C+1}$$

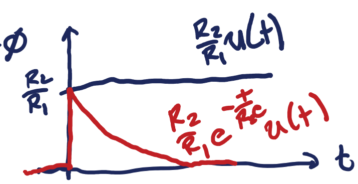
$(-1)^k \frac{R_2}{R_1}$: scale factor; $e^{-sk\frac{T}{2}}$: s-domain time delay

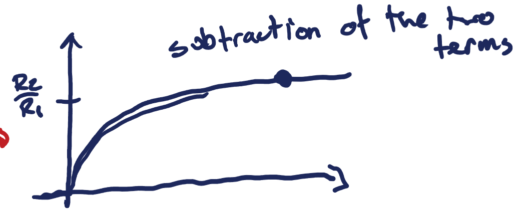
$$\frac{1}{s}\frac{1}{sR_2C+1} = \frac{1}{s} + \frac{-R_2C}{sR_2C+1} = \frac{1}{s} + \frac{-1}{s + \frac{1}{R_2C}}$$

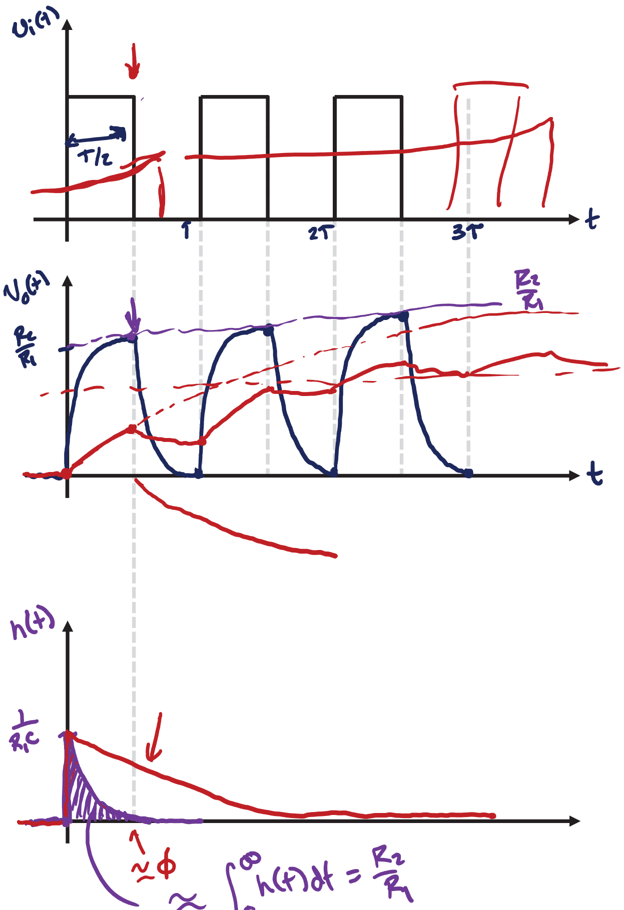
$$v_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{R_2}{R_1} \left(u(t-k\tfrac{T}{2}) - e^{-\frac{t-k\frac{T}{2}}{R_2C}} u(t-k\tfrac{T}{2}) \right) = \sum_{k=0}^{\infty} (-1)^k \frac{R_2}{R_1} \left(1 - e^{-\frac{t-kT/2}{R_2C}}\right) u(t-k\tfrac{T}{2})$$

for $k = \emptyset$







$$v_o(t) = \sum_{k=0}^{\infty} (-1)^k \frac{R_2}{R_1}\left(1 - e^{-\frac{t-kT/2}{R_2C}}\right) u\left(t - k\frac{T}{2}\right)$$

if $\frac{T}{2} \gg R_2C$ → exponential goes through multiple time constants

if $\frac{T}{2} \ll R_2C$ → exponential gets through less than 1 time constant

$\frac{R_2}{R_1}\left(\frac{1}{2}\right)$

$$H(s) = \frac{R_2}{R_1}\frac{1}{sR_2C+1} = \frac{R_2}{R_1}\frac{\frac{1}{R_2C}}{s + \frac{1}{R_2C}}$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \frac{1}{R_1C}e^{-\frac{t}{R_2C}}u(t)$$

$$v_o(t) = \int_0^{\infty} h(t-\tau)\, v_i(\tau)\, d\tau$$