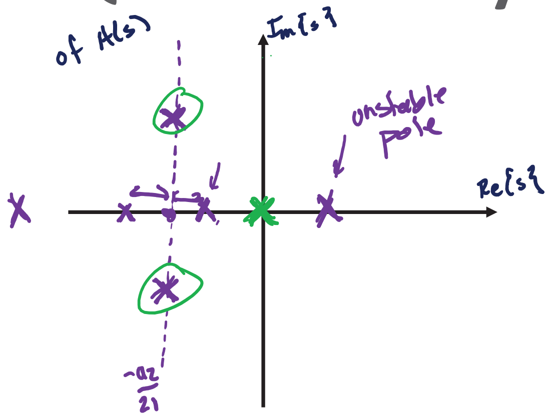
Quiz Takeaways



Linear, time-invariant, causal, real-valued systems

$$v_o(t) = \int_0^\infty h(\tau)\, v_i(t-\tau)\, d\tau$$

$$\Updownarrow$$

$$V_o(s) = H(s) \cdot V_I(s)$$

$$V_o(s) = \left[H_o \frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots} \right] \cdot \left[V_{Io} \frac{(s-z_a)(s-z_b)\cdots}{(s-p_a)(s-p_b)\cdots} \right]$$

All z_i & p_i may be real or complex

Any time a complex pole or zero is present, it is accompanied by its conjugate

$\downarrow$ PFE & $\mathcal{L}^{-1}$

$$v_o(t) = \underbrace{\left[k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots \right]}_{\text{due to poles in } H(s) \text{ Natural response}} + \underbrace{\left[k_a e^{p_a t} + k_b e^{p_b t} + \cdots \right]}_{\text{poles in } V_I(s) \text{ Forced Response}}$$

ex/ step response

$v_i(t) = u(t) \longrightarrow \mathcal{L} \longrightarrow V_I(s) = \frac{1}{s}$

$$H(s) = \frac{A}{a_1 s^2 + a_2 s + a_3}$$

$$V_o(s) = H(s)\, V_I(s)$$

$\rightarrow$ 3 poles at 0, $\frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1}$

from $v_i(t)$ from H(s)

$$\frac{-a_2}{2a_1} \pm \underbrace{\frac{\sqrt{a_2^2 - 4a_1a_3}}{2a_1}}$$

$\leftarrow$ $a_2^2 > 4a_1a_3 \rightarrow$ real

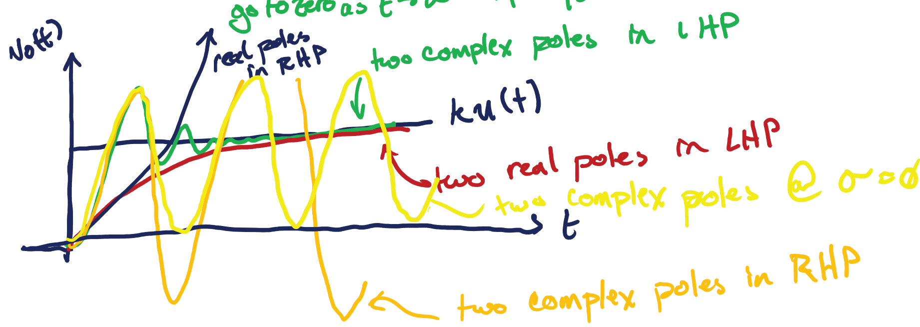
$a_2^2 < 4a_1a_3 \rightarrow$ imaginary

$\frac{-a_2}{2a_1} < \emptyset$

"BIBO" stability

$\rightarrow$ Re{all poles} in open LHP

$$v_o(t) = ku(t) + \underbrace{k_2 e^{p_1 t} + k_3 e^{p_2 t}}_{\text{go to zero as } t \to \infty}$$ if system is stable



$$v_i(t) = \sum_{k=0}^{\infty} u(t-kT) - u\left(t-kT-\tfrac{T}{2}\right)$$

$$\mathcal{L}\{v_i(t)\} = V_i(s) = \sum_{k=0}^{\infty} e^{-skT}\frac{1}{s} - e^{-s\left(kT+\frac{T}{2}\right)}\frac{1}{s}$$

$$= \sum_{k=0}^{\infty} e^{-skT}\frac{1}{s}\underbrace{\left(1-e^{-s\frac{T}{2}}\right)}_{\text{has no "k"}}$$

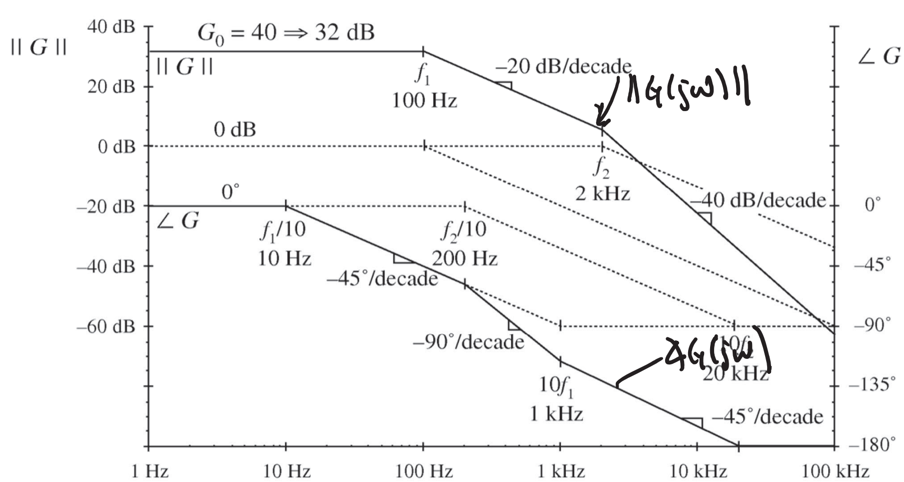
$$= \frac{1}{s}\left(1-e^{-s\frac{T}{2}}\right)\sum_{k=0}^{\infty} e^{-skT} \longrightarrow \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}, \quad \sum_{k=0}^{\infty}\left(e^{-sT}\right)^k = \frac{1}{1-e^{-sT}}$$

$$\boxed{= \frac{1}{s}\,\frac{\left(1-e^{-s\frac{T}{2}}\right)}{1-e^{-sT}}}$$

Example 1

$$G(s) = \frac{G_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$

with $G_0 = 40 \Rightarrow 32\text{ dB}, f_1 = \omega_1/2\pi = 100\text{ Hz}, f_2 = \omega_2/2\pi = 2\text{ kHz}$



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Example 2

$A(j\omega)$

$$A(s) = A_0 \frac{\left(\frac{s}{\omega_1} + 1\right)}{\left(\frac{s}{\omega_2} + 1\right)}$$

Determine the transfer function $A(s)$ corresponding to the following asymptotes: