Example 2

Determine the transfer function $A(s)$ corresponding to the following asymptotes:

\[ A(s) = A_0 \frac{\left( \frac{\omega_1}{\omega_s} + 1 \right) \left( \frac{\omega_2}{\omega_s} + 1 \right)}{\omega_1 \frac{\omega_2}{\omega_s}} = \frac{A_0}{1 + \frac{\omega_1}{\omega_s}} \frac{1 + \frac{\omega_2}{\omega_s}}{1 + \frac{\omega_2}{\omega_s}} \]

Example 3

\[ G(s) = \frac{1}{s} \left( \frac{1}{s} \right) \]

\[ \| G(s) \|_{dB} = 20 \log_{10} \left( \frac{1}{s} \right) - 20 \log_{10} \left( \frac{1}{s} \right) \]

\[ G_x = \omega_1 A_m \]

\[ H(s) = A_m s \left( 1 + \frac{s}{\omega_1} \right) \left( 1 + \frac{s}{\omega_2} \right) \]

Note: Where is DC?  
\[ DC \rightarrow \omega = 0 \]
\[ \log(\omega) \rightarrow -\infty \]
Resonant Poles

Example

\[ G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2LC} \]

Second-order denominator, of the form

\[ G(s) = \frac{1}{1 - a_1s + a_2s^2} \]

with \( a_1 = \frac{L}{R} \) and \( a_2 = LC \)

How should we construct the Bode diagram?

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**Standard Form for Complex Poles**

\[ G(s) = \frac{1}{1 + 2\zeta\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \]

\[ \frac{1}{\omega_0} = \frac{1}{Q\omega_0} = a_1 \]

- When the coefficients of \( s \) are real and positive, then the parameters \( \zeta \), \( \omega_0 \), and \( Q \) are also real and positive.
- The parameters \( \zeta \), \( \omega_0 \), and \( Q \) are found by equating the coefficients of \( s \).
- The parameter \( \omega_0 \) is the angular corner frequency, and we can define \( f_0 = \frac{\omega_0}{2\pi} \).
- The parameter \( \zeta \) is called the *damping factor*. \( \zeta \) controls the shape of the exact curve in the vicinity of \( f = f_0 \). The roots are complex when \( \zeta < 1 \).
- In the alternative form, the parameter \( Q \) is called the *quality factor*. \( Q \) also controls the shape of the exact curve in the vicinity of \( f = f_0 \). The roots are complex when \( Q > 0.5 \).
The Q Factor

In a second-order system, $\zeta$ and $Q$ are related according to

$$Q = \frac{1}{2\zeta}$$

$Q$ is a measure of the dissipation in the system. A more general definition of $Q$, for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{\text{(peak stored energy)}}{\text{(energy dissipated per cycle)}}$$

For a second-order passive system, the two equations above are equivalent. We will see that $Q$ has a simple interpretation in the Bode diagrams of second-order transfer functions.

Magnitude Asymptotes

In the form

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

let $s = j\omega$ and find magnitude:  

$$\|G(j\omega)\| = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}}$$

Asymptotes are

$$\|G\| \to 1 \quad \text{for} \quad \omega << \omega_0$$

$$\|G\| \to \left(\frac{f}{f_0}\right)^{-2} \quad \text{for} \quad \omega >> \omega_0$$
**Exact Magnitude Curve**

\[
\|G(j\omega)\| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}}
\]

At \(\omega = \omega_0\), the exact magnitude is

\[
\|G(j\omega_0)\| = Q
\]

or, in dB:

\[
\|G(j\omega_0)\|_{\text{db}} = 20\log Q
\]

The exact curve has magnitude \(Q\) at \(f = f_0\). The deviation of the exact curve from the asymptotes is \(\|Q\|_{\text{db}}\).  

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**Curves for Varying Q**

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*Fundamentals of Power Electronics* 43  *Chapter 8: Converter Transfer Functions*
Asymptotes for Complex Poles, $Q > 0.5$

Magnitude

Phase

-40 dB/decade

$\| G \|$

Low $Q$ Factorization

Given a second-order denominator polynomial, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2}$$

or

$$G(s) = \frac{1}{1 + \frac{s}{Q \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

When the roots are real, i.e., when $Q < 0.5$, then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

This is a particularly desirable approach when $Q \ll 0.5$, i.e., when the corner frequencies $\omega_1$ and $\omega_2$ are well separated.

$$\omega_1 = \frac{\omega_0}{Q} \frac{1 - \sqrt{1 - 4Q^2}}{2}$$

$$\omega_2 = \frac{\omega_0}{Q} \frac{1 + \sqrt{1 - 4Q^2}}{2}$$
Corner Frequency $\omega_1$

$$\omega_1 = \omega_0 \frac{1 - \sqrt{1 - 4Q^2}}{2}$$

can be written in the form

$$\omega_1 = Q \omega_0 \frac{\omega_0}{F(Q)}$$

where

$$F(Q) = \frac{1}{2} \left( 1 + \sqrt{1 - 4Q^2} \right)$$

For small $Q$, $F(Q)$ tends to 1. We then obtain

$$\omega_1 = Q \omega_0 \quad \text{for} \quad Q \ll \frac{1}{2}$$

For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.

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The Low-Q Approximation

$$\| G \|_{dB}$$

$$f_1 = \frac{Qf_0}{F(Q)}$$

$$\approx Qf_0$$

$$f_2 = \frac{f_0F(Q)}{Q}$$

$$\approx \frac{f_0}{Q}$$

Can be used only if $Q < 0.5$