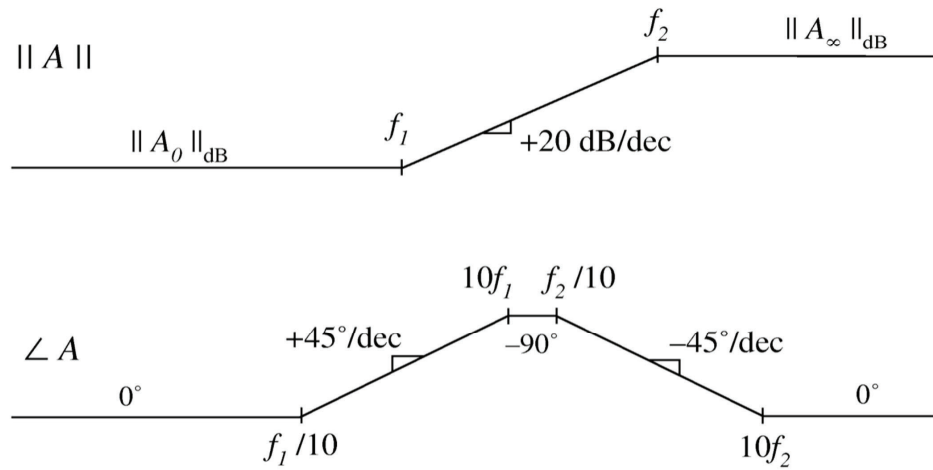


Example 2

Determine the transfer function $A(s)$ corresponding to the following asymptotes:



Example 3

Resonant Poles

Example

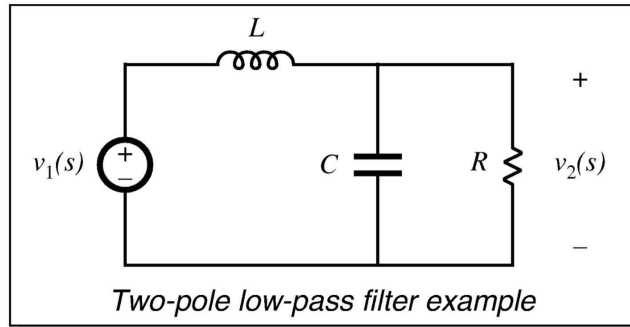
$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2LC}$$

Second-order denominator, of the form

$$G(s) = \frac{1}{1 + a_1s + a_2s^2}$$

with $a_1 = L/R$ and $a_2 = LC$

How should we construct the Bode diagram?



Standard Form for Complex Poles

$$G(s) = \frac{1}{1 + 2\zeta \frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

- When the coefficients of s are real and positive, then the parameters ζ , ω_0 , and Q are also real and positive
- The parameters ζ , ω_0 , and Q are found by equating the coefficients of s
- The parameter ω_0 is the angular corner frequency, and we can define $f_0 = \omega_0/2\pi$
- The parameter ζ is called the *damping factor*. ζ controls the shape of the exact curve in the vicinity of $f=f_0$. The roots are complex when $\zeta < 1$.
- In the alternative form, the parameter Q is called the *quality factor*. Q also controls the shape of the exact curve in the vicinity of $f=f_0$. The roots are complex when $Q > 0.5$.

The Q Factor

In a second-order system, ζ and Q are related according to

$$Q = \frac{1}{2\zeta}$$

Q is a measure of the dissipation in the system. A more general definition of Q , for sinusoidal excitation of a passive element or system is

$$Q = 2\pi \frac{(\text{peak stored energy})}{(\text{energy dissipated per cycle})}$$

For a second-order passive system, the two equations above are equivalent. We will see that Q has a simple interpretation in the Bode diagrams of second-order transfer functions.

Magnitude Asymptotes

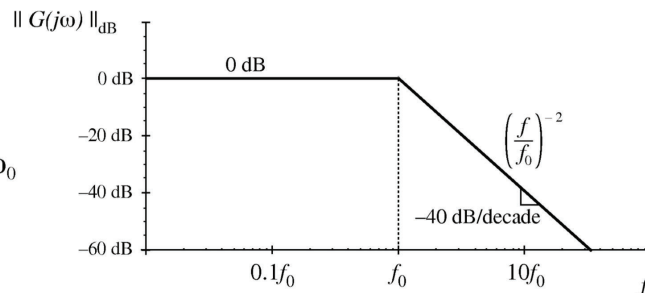
In the form
$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

let $s = j\omega$ and find magnitude:
$$\|G(j\omega)\| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}}$$

Asymptotes are

$$\|G\| \rightarrow 1 \quad \text{for } \omega \ll \omega_0$$

$$\|G\| \rightarrow \left(\frac{f}{f_0}\right)^{-2} \quad \text{for } \omega \gg \omega_0$$



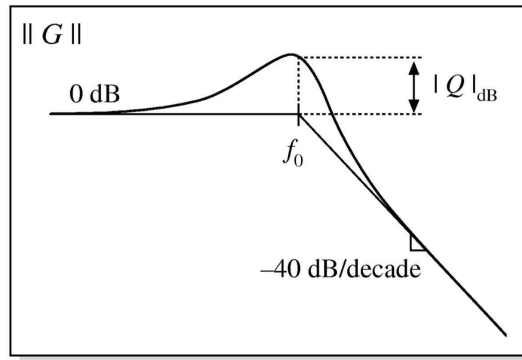
Exact Magnitude Curve

$$\|G(j\omega)\| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}}$$

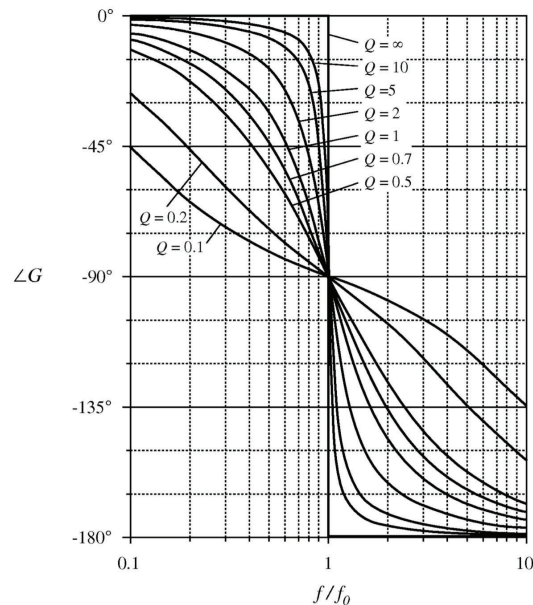
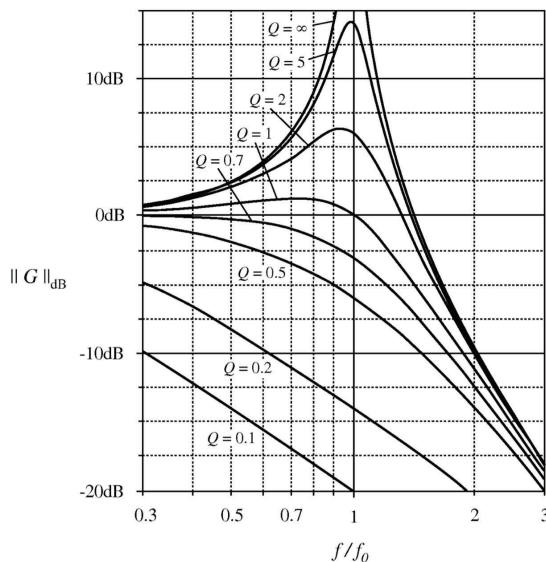
At $\omega = \omega_0$, the exact magnitude is

$$\|G(j\omega_0)\| = Q \quad \text{or, in dB:} \quad \|G(j\omega_0)\|_{\text{dB}} = |Q|_{\text{dB}}$$

The exact curve has magnitude Q at $f = f_0$. The deviation of the exact curve from the asymptotes is $|Q|_{\text{dB}}$

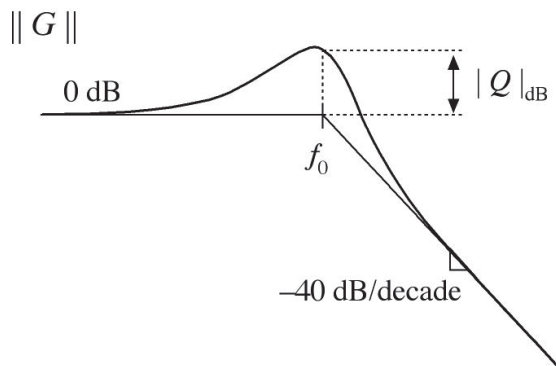


Curves for Varying Q

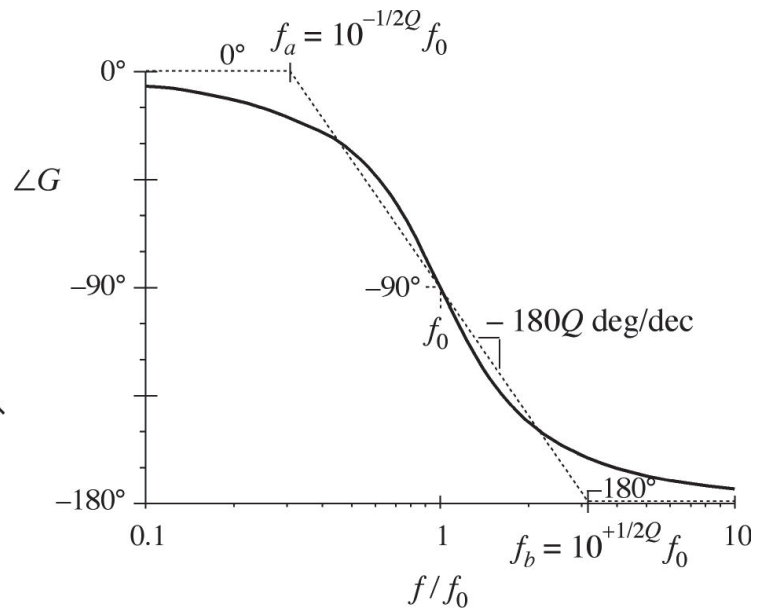


Asymptotes for Complex Poles, $Q > 0.5$

Magnitude



Phase



Low Q Factorization

Given a second-order denominator polynomial, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \quad \text{or} \quad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

When the roots are real, i.e., when $Q < 0.5$, then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

This is a particularly desirable approach when $Q \ll 0.5$, i.e., when the corner frequencies ω_1 and ω_2 are well separated.

$$\omega_1 = \frac{\omega_0}{Q} \frac{1 - \sqrt{1 - 4Q^2}}{2} \quad \omega_2 = \frac{\omega_0}{Q} \frac{1 + \sqrt{1 - 4Q^2}}{2}$$

Corner Frequency ω_1

$$\omega_1 = \frac{\omega_0}{Q} \frac{1 - \sqrt{1 - 4Q^2}}{2}$$

can be written in the form

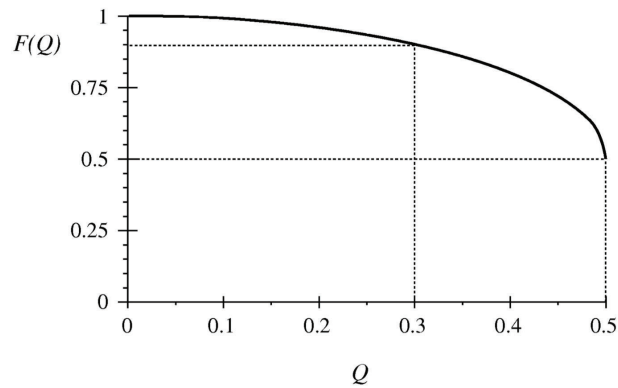
$$\omega_1 = \frac{Q \omega_0}{F(Q)}$$

where

$$F(Q) = \frac{1}{2} (1 + \sqrt{1 - 4Q^2})$$

For small Q , $F(Q)$ tends to 1.
We then obtain

$$\omega_1 \approx Q \omega_0 \quad \text{for } Q \ll \frac{1}{2}$$



For $Q < 0.3$, the approximation $F(Q) = 1$ is within 10% of the exact value.

The Low-Q Approximation

