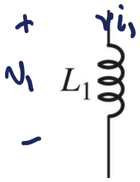
Energy Storage

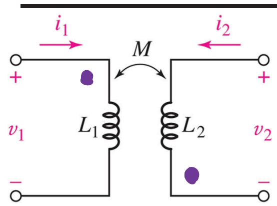


Inductor: if $i_1 = \emptyset$ for $t < \emptyset$, then goes to I_x @ t_x

$$E_x = \int_0^{t_x} P_L(t)\,dt = \int_0^{t_x} v_1(t)\, i_1(t)\,dt = \int_0^{t_x} L_1 i_1(t) \frac{di_1(t)}{dt}\,dt$$

$$= L_1 \int_0^{t_x} \frac{1}{2}\left[\frac{d}{dt} i_1(t)^2\right] dt = \frac{1}{2} L_1 \left[i_1(t)^2 \right] \Big|_{0 = i_1(t=0)}^{I_x = i_1(t=t_x)}$$

$$\boxed{= \frac{1}{2} L_1 I_x^2}$$



prior to time $t=0$

$i_1 = 0$ & $i_2 = 0$

then @ $t = t_x$

$i_1 = I_{x1}$ & $i_2 = I_{x2}$

$$E_x = \int_0^{t_x} (i_1 v_1 + i_2 v_2)\,dt =$$

$$= \int_0^{t_x} \left(i_1 L_1 \frac{di_1}{dt} \pm i_1 M \frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt} \pm i_2 M \frac{di_1}{dt} \right) dt$$

$$= \frac{1}{2} L_1 I_{x1}^2 + \frac{1}{2} L_2 I_{x2}^2 \pm M \int_0^t \underbrace{\left(i_1 \frac{di_2}{dt} + i_2 \frac{di_1}{dt} \right)}_{= \frac{d}{dt}(i_1 \cdot i_2)} dt$$

$$\boxed{E_x = \frac{1}{2} L_1 I_{x1}^2 + \frac{1}{2} L_2 I_x^2 \pm M I_{x1} I_{x2}}$$

↳ depends on dot notation.

$$E_x = \frac{1}{2} L_1 I_{x1}^2 + \frac{1}{2} L_2 I_{x2}^2 - M I_{x1} I_{x2}$$

- Assume inverting winding polarity for the scenario described $E_x \geq \emptyset$ (otherwise its a source)

then

$$E_x = \frac{1}{2} L_1 I_{x1}^2 + \frac{1}{2} L_2 I_{x2}^2 - M I_{x1} I_{x2} \geq \emptyset$$

$$M \leq \frac{\frac{1}{2} L_1 I_{x1}^2 + \frac{1}{2} L_2 I_{x2}^2}{I_{x1} I_{x2}} = \frac{1}{2} L_1 \frac{I_{x1}}{I_{x2}} + \frac{1}{2} L_2 \frac{I_{x2}}{I_{x1}}$$

Must hold $\forall (I_{x1}, I_{x2})$

let's substitute $x = \frac{I_{x1}}{I_{x2}}$

$f(x) = \frac{1}{2} L_1 x + \frac{1}{2} L_2 \frac{1}{x}$, find $x = x_{min}$ to minimize $f(x)$

$$f'(x) = \frac{1}{2} L_1 - \frac{1}{2} L_2 \frac{1}{x^2} = 0 \rightarrow x_{min} = \pm\sqrt{\frac{L_2}{L_1}}$$

$$f''(x) = L_2 \frac{1}{x^3}$$

$$M \leq \frac{1}{2} L_1 \sqrt{\frac{L_2}{L_1}} + \frac{1}{2} L_2 \frac{1}{\sqrt{\frac{L_2}{L_1}}} \longrightarrow \boxed{M \leq \sqrt{L_1 L_2}}$$

Coupling Coefficient

$M \leq \sqrt{L_1 L_2}$ → Must hold for any physically realizable element

one check: let's consider two identical inductors perfectly overlapped

→ $L_1 = L_2 = L$ $\quad M \leq \sqrt{L_1 L_2} = \sqrt{L \cdot L} = L$

justification: recall $L_1 = k_{11} N_1^2$, $L_2 = k_{22} N_2^2$, $M = k_{12} N_1 N_2$

In this perfectly matched case, $L_1 = L_2 = M$

Define $k = \dfrac{M}{\sqrt{L_1 L_2}}$ → "coupling coefficient"

$0 \leq k \leq 1$
- $k = \emptyset$ → two seperate inductors
- ⋮ → partial coupling
- $k = 1$ → perfect coupling → "transformer"

Transformers

special case of coupled inductors where $k \approx 1 \iff M \approx \sqrt{L_1 L_2}$

$$v_1 = L_1 \frac{di_1}{dt} \pm \sqrt{L_1 L_2} \frac{di_2}{dt}$$
$$v_2 = L_2 \frac{di_2}{dt} \pm \sqrt{L_1 L_2} \frac{di_1}{dt}$$

$v_1 = v_2 \left(\dfrac{\sqrt{L_1 L_2}}{L_2} \right)$

$$\boxed{v_1 = v_2 \sqrt{\frac{L_1}{L_2}}}$$

$v_1 = v_2 \sqrt{\dfrac{\cancel{k_{11}} N_1^2}{\cancel{k_{22}} N_2^2}}$ $\quad$ if $k = 1$, $k_{11} = k_{22}$

$$\boxed{v_1 = v_2 \frac{N_1}{N_2}}$$