Announcements

- Experiment 2 (optional) due Friday 4/24
- TNvoice evaluations
  - 5 points EC on final if we get 100% participation
  - 64%, currently; deadline Monday 4/27
- Quiz #4 and Quiz #5 graded
- Today is final day to change grading option for all courses

A-F	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
S/NC				S						NC		
S/CR/NC				S						CR		NC

Slido: #Q711

Chapter 17

FOURIER ANALYSIS

Fourier Series Representation

$f(t+T_0) = f(t)$

Assume we have some function $f(t)$ which is periodic with period $T_0 = \frac{2\pi}{\omega_0}$

$$\boxed{f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)}$$

$f(t)$ can be expressed this way if

1. $f(t)$ is single-valued
2. $\int_{t_0}^{t_0+T_0} |f(t)|\, dt$ exists
3. $f(t)$ had finite discontinuities and max/min per period

DC term = Average value of $f(t)$ $\quad a_0 = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)\,dt$

Alternate forms

$$f(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k) \quad \begin{cases} A_k = \sqrt{{a_k}^2 + {b_k}^2} \\ \varphi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right) \end{cases}$$

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \quad \begin{cases} c_k = \frac{1}{2}(a_k - jb_k) \\ c_{-k} = \frac{1}{2}(a_k + jb_k) \\ c_0 = a_0 \end{cases}$$

Fourier Series Coefficients

Let's try calculating the average value of $f(t)\cdot\cos(n\omega_0 t)$ $\quad n \in \mathbb{Z}$

$$= \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)\cos(n\omega_0 t)\,dt = \frac{1}{T_0}\int_{t_0}^{t_0+T_0}\left[a_0 + \left(\sum_{k=1}^{\infty} a_k\cos(k\omega_0 t) + b_k\sin(k\omega_0 t)\right)\cos(n\omega_0 t)\right]dt$$

$$= \frac{a_0}{T_0}\underbrace{\int_{t_0}^{t_0+T_0}\cos(n\omega_0 t)\,dt}_{\phi} + \sum_{k=1}^{\infty}\frac{1}{T_0}\int_{t_0}^{t_0+T_0} a_k \underbrace{\cos(k\omega_0 t)\cos(n\omega_0 t)}_{\frac{\cos((k+n)\omega_0 t) + \cos((k-n)\omega_0 t)}{2}}\,dt + \frac{1}{T_0}\underbrace{\int_{t_0}^{t_0+T_0} b_k\sin(k\omega_0 t)\cos(n\omega_0 t)\,dt}_{\phi}$$

$$= \frac{1}{2}\left[\sin((k+n)\omega_0 t) + \sin((k-n)\omega_0 t)\right]$$

$$= \begin{cases} \phi, & k \neq n \\ \frac{1}{2}, & k = n \end{cases}$$

$$\boxed{\frac{2}{T_0}\int_{t_0}^{t_0+T_0} f(t)\cos(n\omega_0 t)\,dt = a_n}$$

for $k=n$

$$a_k = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} f(t)\cos(k\omega_0 t)\,dt \qquad b_k = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} f(t)\sin(k\omega_0 t)\,dt \qquad c_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)e^{-jk\omega_0 t}\,dt$$

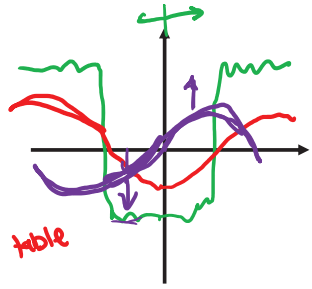
Symmetry in Fourier Series

Even functions

$f(t) = f(-t)$

$b_k = 0$

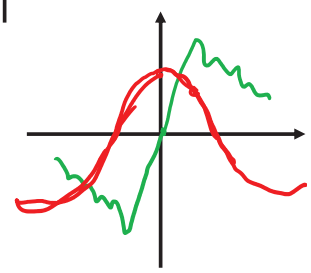
typo in book table



Odd functions

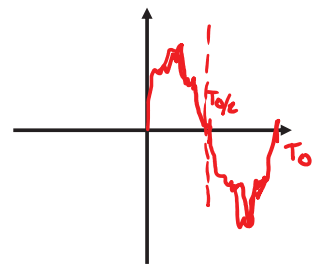
$f(t) = -f(-t)$

$a_k = 0$



Half-wave symmetric functions

$f(t + \frac{T_0}{2}) = -f(t)$



$a_k,\ b_k = 0$ for even k

Non-periodic Waveforms: Fourier Transform (L19)

Non-periodic waveform is "say" it repeats after period $T \to \infty$

$\omega_0 = \frac{2\pi}{T} \to \phi$

Periodic: $C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$

Non-periodic: $TC_n = \boxed{\int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = F(\omega)}$

looks just like Laplace but with $\sigma = \phi$ or $s = j\omega$

Fourier Transform

Periodic: $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

Non-periodic $\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega}$

Careful: Laplace usually unilateral, Fourier always bilateral

$\mathcal{L}\{\sin(\omega t)\} = ?$

$\mathcal{L}\{\sin(\omega t)u(t)\} = \frac{\omega}{s^2+\omega^2}$